REDSPOT

IGCSE

(Syllabus 0580)

MATHEMATICS

Paper 2 (Extended) - All Variants

(Topical)

period	2018 to June-2024
contents	June & November, Paper 2 (P21, P22 & P23) Worked Solutions
form	Topic By Topic
compiled for	IGCSE

Tel No : 042-35201010

Mobile No : 0300-8447654

E-Mail : info@redspot.com.pk

Website : www.redspot.pk

Address : P.O. Box 5041, Model Town, Lahore, Pakistan.

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TOPIC 1

Numbers

1. Calculate.

$$\frac{5.38 - 0.98}{0.743 - 0.343}$$

.....[1]

[Nov/2018/P23/Q4][Note: Question is modified to solve without a calculator]

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2. **Without using a calculator**, work out $\frac{1}{4} \div \frac{2}{3}$.

You must show all your working and give your answer as a fraction.

..... [2]

[Nov/2018/P23/Q9]

---

3. Change the recurring decimal 0.18 to a fraction.
You must show all your working.

..... [2]

[Nov/2018/P23/Q13]

---

4. Calculate $\sqrt[3]{8.2^2 - 3.24}$

..... [1]

[June/2019/P21/Q3][Note: Question is modified to solve without a calculator]

---

5. Write the recurring decimal $0.4\dot{7}$ as a fraction.
Show all your working.

..... [2]

[June/2019/P21/Q9]

---

6. 27 28 29 30 31 32 33

From the list of numbers, write down

(a) a multiple of 7,

..... [1]

(b) a cube number,

..... [1]

(c) a prime number.

..... [1]

[June/2019/P21/Q12]

---

7. Work out $\frac{5}{6}+\frac{2}{3}$. You must show all your working and give your answer as a mixed number in its simplest form.

..... [3]

[June/2019/P21/Q14]

---

8. Write down a prime number between 50 and 60.

..... [1]

[June/2019/P22/Q1]

---

9. Write the recurring decimal $0.\dot{7}$ as a fraction.

..... [1]

[June/2019/P22/Q3]

---

10. Find the highest common factor (HCF) of 90 and 48.

..... [2]

[June/2019/P22/Q9]

---

11. Work out $2\frac{1}{4} \div \frac{3}{7}$. You must show all your working and give your answer as a mixed number in its simplest form.

..... [3]

[June/2019/P22/Q13]

---

12. Calculate.

(a) $-12 \div -2$

..... [1]

(b) $\sqrt[3]{2^4 + 11}$

..... [1]

[June/2019/P23/Q6] [Note: Part (b) is modified to solve without a calculator]

---

13. Here is a list of numbers.

$$21 \qquad \frac{2}{3} \qquad \sqrt{13} \qquad 31 \qquad \sqrt{121} \qquad 51 \qquad 0.7$$

From this list, write down

(a) a prime number,

..... [1]

(b) an irrational number.

..... [1]

[June/2019/P23/Q7]

---

14. Work out $\frac{12}{35} \times \frac{7}{9}$. You must show all your working and give your answer as a fraction in its simplest form.

..... [2]

[June/2019/P23/Q9]

ANSWERS

Topic 1 - Numbers

1. $\dfrac{5.38-0.98}{0.743-0.343}$

$=\dfrac{4.4}{0.4}\times\dfrac{10}{10}$

$=\dfrac{44}{4}=11$

2. $\dfrac{1}{4}\div\dfrac{2}{3}$

$=\dfrac{1}{4}\times\dfrac{3}{2}=\dfrac{3}{8}$

3. Let $x=0.1\dot{8}$

$\Rightarrow \quad x=0.188888$

Multiply by 10,

$\Rightarrow \quad 10x=1.88888$ ..... (1)

Multiply by 100,

$\Rightarrow \quad 100x=18.8888$ ..... (2)

Subtract (1) from (2)

$$\begin{array}{r} 100x=18.8888... \\ 10x=\ 1.8888... \\ \hline 90x\ =17 \end{array}$$

$\Rightarrow \quad x=\dfrac{17}{90}$

4. $\sqrt[3]{8.2^2-3.24}$

$=\sqrt[3]{67.24-3.24}$

$=\sqrt[3]{64}=\sqrt[3]{4^3}=4$

5. Let $x=0.4\dot{7}$

$\Rightarrow \quad x=0.477777$

Multiply by 10,

$\Rightarrow \quad 10x=4.77777....$ (1)

Multiply by 100,

$\Rightarrow \quad 100x=47.77777....$ (2)

Subtract (1) from (2)

$$\begin{array}{r} 100x=47.77777... \\ 10x=\ 4.77777... \\ \hline 90x\ =43 \end{array}$$

$\Rightarrow \quad x=\dfrac{43}{90}$

6. **(a)** Multiple of 7 = 28

(b) Cube number = 27

(c) Prime number = 29 or 31

7. $\dfrac{5}{6}+\dfrac{2}{3}$

$=\dfrac{5+4}{6}$

$=\dfrac{9}{6}=\dfrac{3}{2}=1\dfrac{1}{2}$

8. 53 or 59

9. Let $x=0.\dot{7}$

$\Rightarrow \quad x=0.77777....$ (1)

$\Rightarrow \quad 10x=7.77777....$ (2)

Subtract (1) from (2)

$$\begin{array}{r} 10x=7.77777... \\ x=0.77777... \\ \hline 9x\ =7 \end{array}$$

$\Rightarrow \quad x=\dfrac{7}{9}$

10. $90=2\times3\times3\times5$

$48=2\times2\times2\times2\times3$

$\therefore$ HCF $=2\times3=6$

11. $2\dfrac{1}{4}\div\dfrac{3}{7}$

$=\dfrac{9}{4}\times\dfrac{7}{3}$

$=\dfrac{21}{4}=5\dfrac{1}{4}$

12. **(a)** $-12\div-2$

$=\dfrac{-12}{-2}=6$

(b) $\sqrt[3]{2^4+11}$

$=\sqrt[3]{16+11}$

$=\sqrt[3]{27}=\sqrt[3]{3^3}=3$

13. **(a)** Prime number = 31 or $\sqrt{121}$

(b) Irrational number = $\sqrt{13}$

TOPIC 14

Solutions of Equations

1. Solve the equation. $9f + 11 = 3f + 23$

$f =$ ..... [2]

[June/2019/P22/Q6]

---

2. Solve the simultaneous equations. You must show all your working.

$$5x + 8y = 4$$
$$\frac{1}{2}x + 3y = 7$$

$x =$ .....

$y =$ .....

[3]

[June/2019/P22/Q14]

---

3. Rearrange this formula to make m the subject.

$$P = \frac{k + m}{m}$$

..... [4]

[June/2019/P22/Q19]

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4. Rearrange $2(w+h)=P$ to make w the subject.

w = ... [2]

[June/2019/P23/Q10]

---

5. Complete this statement with an expression in terms of m.

$$18m^3+9m^2+14m+7 \;=\; (9m^2+7)(\ldots\ldots\ldots\ldots)$$

[2]

[June/2019/P23/Q11]

---

6. One solution of the equation $ax^2+a=150$ is $x=7$.

(a) Find the value of a.

a = ... [2]

(b) Find the other solution.

x = ... [1]

[June/2019/P23/Q14]

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7. Solve. $\frac{x-2}{3}=3$

$x =$ ..... [2]

[Nov/2019/P21/Q6]

---

8. $P=2r+\pi r$

Rearrange the formula to write r in terms of P and π.

$r =$ ..... [2]

[Nov/2019/P21/Q11]

---

9. $y=mx+c$

Find the value of y when $m=-3$, $x=-2$ and $c=-8$.

$y =$ ..... [2]

[June/2020/P22/Q5]

---

10. Solve the equation. $\frac{1-x}{3}=5$

$x =$ ..... [2]

[June/2020/P22/Q14]

---

11. Make y the subject of the formula, $h^2=x^2+2y^2$

$y =$ ..... [3]

[June/2020/P22/Q19]

---

12. (a) Write $x^2 - 18x - 40$ in the form $(x+k)^2 + h$.

..... [2]

(b) Use your answer to **part (a)** to solve the equation $x^2 - 18x - 40 = 0$

x = ... or x = ... [2]

[June/2020/P23/Q18] [Note: Question is modified to solve without a calculator]

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13. Make x the subject of this formula.

$$2y = 5x - 7$$

x = ... [2]

[Nov/2020/P21/Q7]

---

14. Solve the equation. $6 - 2x = 3x$

x = ... [2]

[Nov/2020/P22/Q3]

---

15. Solve the simultaneous equations.

$$2x + y = 7$$
$$3x - y = 8$$

$x =$ .....

$y =$ .....

[2]

[Nov/2020/P22/Q9]

---

16. Solve the simultaneous equations. You must show all your working.

$$3x - 8y = 22$$
$$x + 4y = 4$$

$x =$ .....

$y =$ .....

[3]

[Nov/2020/P23/Q10]

---

17. $m = 2p + \sqrt{\frac{x}{y}}$

Make x the subject of this formula.

$x =$ ..... [3]

[Nov/2020/P23/Q15]

ANSWERS

Topic 14 - Solutions of Equations

1. $9f+11=3f+23$

$9f-3f=23-11$

$6f=12 \Rightarrow f=\frac{12}{6}=2$

2. $5x+8y=4$ ..... (1)

$\frac{1}{2}x+3y=7$ ..... (2)

Solving the equations simultaneously,

eq. (1): $5x + 8y = 4$

eq. (2)$\times 10$: $5x + 30y = 70$

$-22y=-66$

$\Rightarrow y=\frac{-66}{-22}=3$

Substitute $y=3$ into eq. (1)

$5x+8(3)=4$

$\Rightarrow 5x+24=4 \Rightarrow 5x=-20 \Rightarrow x=-4$

$\therefore x=-4, \quad y=3$

3. $P=\frac{k+m}{m}$

$Pm=k+m$

$Pm-m=k$

$m(P-1)=k$

$m=\frac{k}{P-1}$

4. $2(w+h)=P$

$w+h=\frac{P}{2} \Rightarrow w=\frac{P}{2}-h$

5. $\text{LHS}=18m^3+9m^2+14m+7$

$=18m^3+14m+9m^2+7$

$=2m(9m^2+7)+1(9m^2+7)$

$=(9m^2+7)(2m+1)$

$\therefore$ Required expression is, $2m+1$

6. **(a)** $ax^2+a=150$

Substitute $x=7$,

$\Rightarrow a(7)^2+a=150$

$\Rightarrow 49a+a=150$

$\Rightarrow 50a=150 \Rightarrow a=\frac{150}{50}=3$

(b) $ax^2+a=150$

Substitute $a=3$,

$\Rightarrow 3x^2+3=150$

$\Rightarrow 3x^2=147$

$\Rightarrow x^2=\frac{147}{3}$

$\Rightarrow x^2=49 \Rightarrow x=\pm 7$

$\therefore$ Other solution is, $x=-7$

7. $\frac{x-2}{3}=3$

$x-2=9$

$x=9+2=11$

8. $P=2r+\pi r$

$\Rightarrow 2r+\pi r=P$

$\Rightarrow r(2+\pi)=P \Rightarrow r=\frac{P}{2+\pi}$

9. $y=mx+c$

$y=(-3)(-2)+(-8)$

$y=6-8=-2$

10. $\frac{1-x}{3}=5$

$1-x=15$

$-x=14 \Rightarrow x=-14$

11. $h^2=x^2+2y^2$

$\Rightarrow x^2+2y^2=h^2$

$\Rightarrow 2y^2=h^2-x^2$

$\Rightarrow y^2=\frac{h^2-x^2}{2} \Rightarrow y=\pm\sqrt{\frac{h^2-x^2}{2}}$

12. **(a)** $x^2-18x-40$

Applying completing the square method,

$=x^2-2(x)(9)+(9)^2-(9)^2-40$

$=(x-9)^2-81-40$

$=(x-9)^2-121$

(b) $x^2 - 18x - 40 = 0$

$\Rightarrow \quad (x-9)^2 - 121 = 0$

$\Rightarrow \quad (x-9)^2 = 121$

$\Rightarrow \quad x - 9 = \pm\sqrt{121}$

$\Rightarrow \quad x - 9 = \pm 11$

$\Rightarrow \quad x - 9 = 11 \quad \text{or} \quad x - 9 = -11$

$\therefore \quad x = 20 \text{ or } x = -2$

13. $2y = 5x - 7$

$\Rightarrow \quad 5x = 2y + 7 \quad \Rightarrow \quad x = \dfrac{2y+7}{5}$

14. $6 - 2x = 3x$

$6 = 3x + 2x$

$6 = 5x \quad \Rightarrow \quad x = \dfrac{6}{5} = 1.2$

15. $2x + y = 7 \quad \Rightarrow \quad y = 7 - 2x$ ..... (1)

$3x - y = 8$ ..... (2)

Substitute eq. (1) into eq. (2),

$3x - (7 - 2x) = 8$

$\Rightarrow \quad 3x - 7 + 2x = 8$

$\Rightarrow \quad 5x = 15 \quad \Rightarrow \quad x = 3$

Substitute $x = 3$ into eq. (1), $y = 7 - 2(3) = 1$

$\therefore \quad x = 3, \quad y = 1$

16. $3x - 8y = 22$ ..... (1)

$x + 4y = 4 \quad \Rightarrow \quad x = 4 - 4y$ ..... (2)

Substitute eq. (2) into eq. (1),

$3(4 - 4y) - 8y = 22$

$\Rightarrow \quad 12 - 12y - 8y = 22$

$\Rightarrow \quad -20y = 10 \quad \Rightarrow \quad y = \dfrac{10}{-20} = -\dfrac{1}{2}$

Substitute $y = -\dfrac{1}{2}$ into eq. (2),

$x = 4 - 4\left(-\dfrac{1}{2}\right) \quad \Rightarrow \quad x = 4 + 2 = 6$

$\therefore \quad x = 6, \quad y = -\dfrac{1}{2}$

17. $m = 2p + \sqrt{\dfrac{x}{y}}$

$\Rightarrow \quad \sqrt{\dfrac{x}{y}} = m - 2p$

$\Rightarrow \quad \dfrac{x}{y} = (m - 2p)^2$

$\Rightarrow \quad x = y(m - 2p)^2$

18. $2x + y = 3$ ..... (1)

$x - 5y = 40$ ..... (2)

Solving the equations simultaneously,

eq. (1) × 5 : $10x + 5y = 15$

eq. (2) : $x - 5y = 40$

$11x = 55$

$\Rightarrow \quad x = \dfrac{55}{11} = 5$

Substitute $x = 5$ into eq. (1)

$2(5) + y = 3 \quad \Rightarrow \quad 10 + y = 3 \quad \Rightarrow \quad y = -7$

$\therefore \quad x = 5, \quad y = -7$

19. $2mh = g(1 - h)$

$\Rightarrow \quad 2mh = g - gh$

$\Rightarrow \quad 2mh + gh = g$

$\Rightarrow \quad h(2m + g) = g \quad \Rightarrow \quad h = \dfrac{g}{2m+g}$

20. $a = \dfrac{b^2}{5c}$

$\Rightarrow \quad 4.9 = \dfrac{b^2}{5(2)}$

$\Rightarrow \quad \dfrac{b^2}{10} = 4.9$

$\Rightarrow \quad b^2 = 49$

$\Rightarrow \quad b = \pm\sqrt{49} = \pm 7$

21. $4x - 2y = -13$ ..... (1)

$-3x + 4y = 11$ ..... (2)

Solving the equations simultaneously,

eq. (1) × 2 : $8x - 4y = -26$

eq. (2) : $-3x + 4y = 11$

$5x = -15$

$\Rightarrow \quad x = \dfrac{-15}{5} = -3$

Substitute $x = -3$ into eq. (2)

$-3(-3) + 4y = 11$

$\Rightarrow \quad 9 + 4y = 11 \quad \Rightarrow \quad 4y = 2 \quad \Rightarrow \quad y = \dfrac{1}{2}$

$\therefore \quad x = -3, \quad y = \dfrac{1}{2}$

22. $y = \dfrac{3x - 2}{1 - x}$

$\Rightarrow \quad y(1 - x) = 3x - 2$

$\Rightarrow \quad y - xy = 3x - 2$

$\Rightarrow \quad y + 2 = 3x + xy$

$\Rightarrow \quad y + 2 = x(3 + y) \quad \Rightarrow \quad x = \dfrac{y+2}{3+y}$

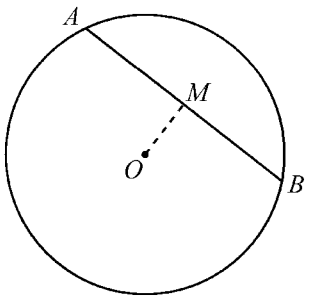
TOPIC 27

Circle Theorems

1. The diagram shows a circle, centre O.

AB is a chord of length 12 cm.
M is the mid-point of AB and $OM = 4$ cm.

Calculate the radius of the circle. Give your answer in the form $\sqrt{q}$.



..... cm [3]

[June/2018/P22/Q16] [Note: Question is modified to solve without a calculator]

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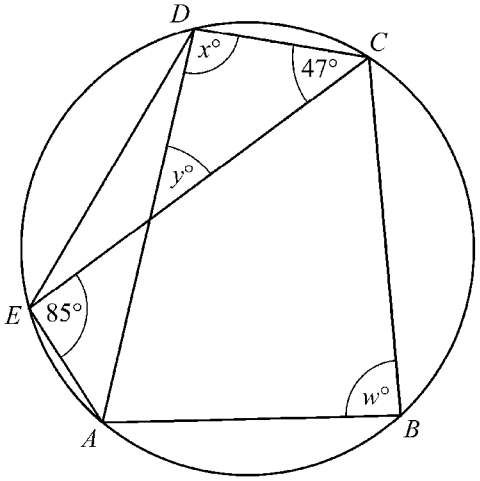
2. The points A, B, C, D and E lie on the circumference of the circle.

Angle $DCE = 47°$ and angle $CEA = 85°$.

Find the values of w, x and y.



w = ...

x = ...

y = ...

[3]

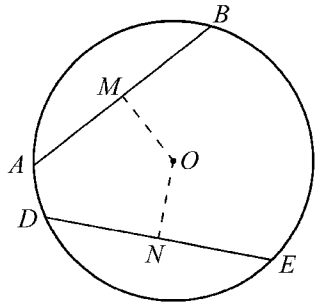
[June/2018/P23/Q20]

---

3. The diagram shows a circle, centre O.

AB and DE are chords of the circle.
M is the mid-point of AB and N is the mid-point of DE.
$AB = DE = 9$ cm and $OM = 5$ cm.

Find ON.



$ON =$ ..... cm [1]

[Nov/2018/P22/Q2]

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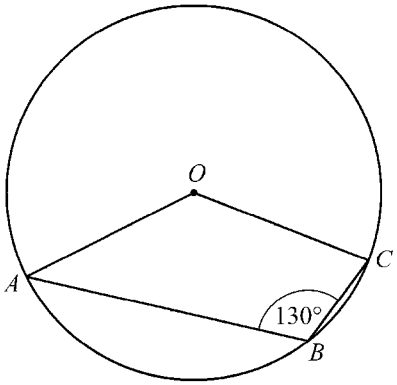
4. A, B and C are points on the circle, centre O.

Find the obtuse angle AOC.



Angle $AOC =$ ..... [2]

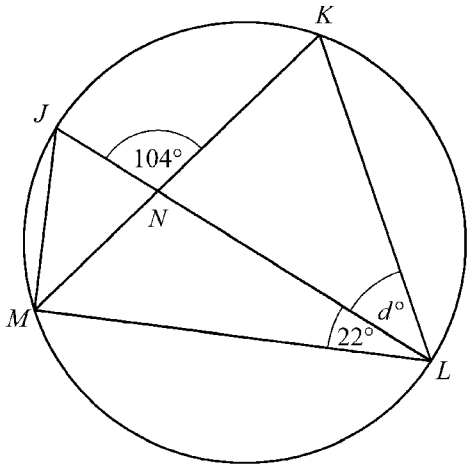
[June/2019/P21/Q8]

---

5. J, K, L and M are points on the circumference of a circle with diameter JL.

JL and KM intersect at N.
Angle $JNK = 104°$ and angle $MLJ = 22°$.

Work out the value of d.



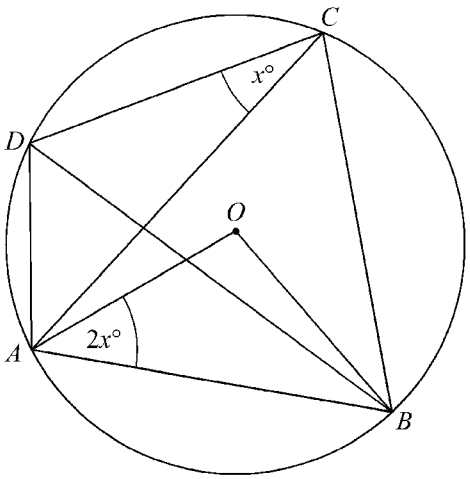
$d =$ ..... [4]

[June/2019/P23/Q19]

6. In the diagram, A, B, C and D lie on the circumference of a circle, centre O.

Angle $ACD = x°$ and angle $OAB = 2x°$.

Find an expression, in terms of x, in its simplest form for



(a) angle AOB,

Angle AOB = ... [1]

(b) angle ACB,

Angle ACB = ... [1]

(c) angle DAB.

Angle DAB = ... [2]

[Nov/2019/P22/Q19]

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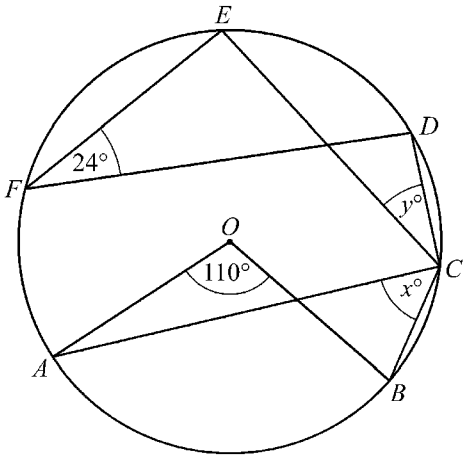
7. Points A, B, C, D, E and F lie on the circle, centre O.

Find the value of x and the value of y. [2]



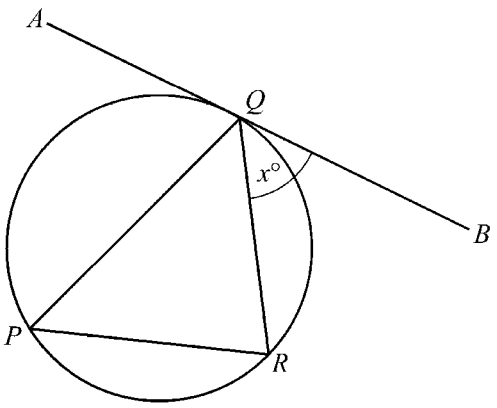
x = ..., y = ...

[June/2020/P21/Q10]

8. P, R and Q are points on the circle.

AB is a tangent to the circle at Q.
QR bisects angle PQB.
Angle $BQR = x°$ and $x < 60$.

Use this information to show that triangle PQR is an isosceles triangle.
Give a geometrical reason for each step of your work.



[3]

[June/2020/P21/Q15]

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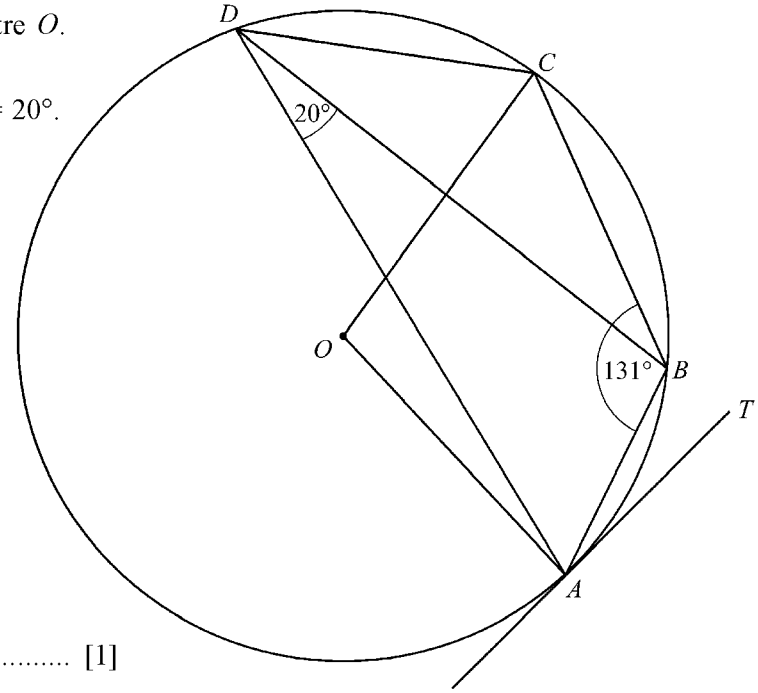
9. A, B, C and D lie on the circle, centre O.

TA is a tangent to the circle at A.
Angle $ABC = 131°$ and angle $ADB = 20°$.

Find

(a) angle ADC,



Angle ADC = ... [1]

(b) angle AOC,

Angle AOC = ... [1]

(c) angle BAT,

Angle BAT = ... [1]

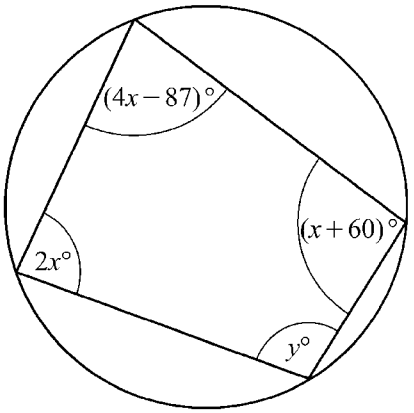
(d) angle OAB.

Angle OAB = ... [1]

[June/2020/P22/Q20]

10. The diagram shows a cyclic quadrilateral.

Find the value of y.



y = ... [4]

[Nov/2020/P21/Q20]

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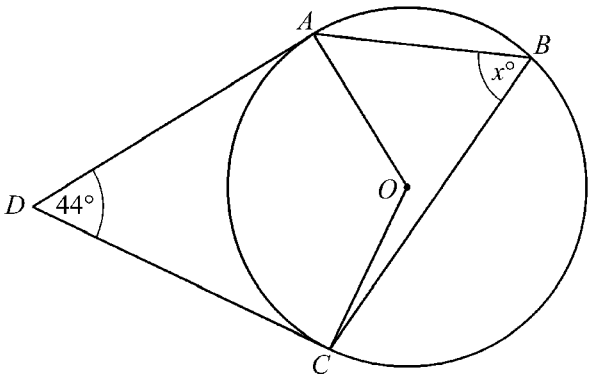
11. A, B and C are points on a circle, centre O.

DA and DC are tangents.
Angle $ADC = 44°$.

Work out the value of x.



x = ... [3]

[June/2021/P22/Q13]

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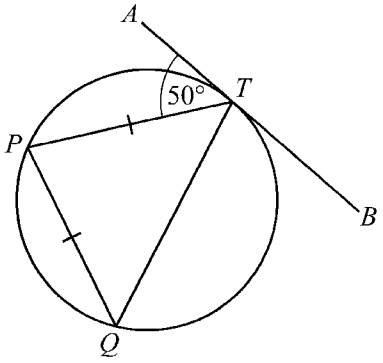
12. (a) P, Q and T are points on a circle.

ATB is a tangent to the circle at T and $PT = PQ$.

Find angle TPQ.



Angle TPQ = ... [2]

ANSWERS

Topic 27 - Circle Theorems

1. $AM = \frac{12}{2} = 6$ cm

In ΔOAM, using Pythagoras Theorem,

$OA = \sqrt{OM^2 + AM^2}$

$\Rightarrow \quad OA = \sqrt{4^2 + 6^2}$

$\Rightarrow \quad OA = \sqrt{16 + 36} = \sqrt{52}$

$\therefore$ Radius of the circle $= \sqrt{52}$ cm.

2. $ABCE$ is a cyclic quadrilateral,

$\therefore \quad w° + 85° = 180°$ (opp. $\angle$s of a cyclic quad. are supplementary)

$\Rightarrow \quad w° = 180° - 85°$

$= 95°$

$ABCD$ is a cyclic quadrilateral,

$\therefore \quad x° + w° = 180°$

$\Rightarrow \quad x° + 95° = 180°$

$\Rightarrow \quad x° = 180° - 95°$

$= 85°$

$x° + y° + 47° = 180°$ ($\angle$ sum of a triangle is 180°)

$\Rightarrow \quad 85° + y° + 47° = 180°$

$\Rightarrow \quad y° = 180° - 132°$

$= 48°$

> Note:
> $x°$ can also be found as below.
> $A\hat{D}C = A\hat{E}C$ (angles in the same segment)
> $\Rightarrow \quad x° = 85°$

3. $ON = 5$ cm (equal chords are equidistant from centre).

4. Reflex $A\hat{O}C = 2(130°) = 260°$

($\angle$ at centre is $2 \times \angle$ at circumference)

$\therefore$ Obtuse $A\hat{O}C = 360° - 260°$

$= 100°$

5. In ΔJML,

$J\hat{M}L = 90°$ (right angle in semicircle)

$M\hat{J}L + 90° + 22° = 180°$ (angle sum of a Δ is 180°)

$\Rightarrow \quad M\hat{J}L = 180° - 90° - 22°$

$\Rightarrow \quad M\hat{J}L = 68°$

$M\hat{K}L = M\hat{J}L$ (angles in the same segment)

$\Rightarrow \quad M\hat{K}L = 68°$

Now, in ΔKLN,

$d° + 68° = 104°$ (ext. angle of Δ = sum of opp. interior angles)

$\Rightarrow \quad d° = 104° - 68°$

$= 36°$

6. **(a)** ΔOAB is isosceles with $OA = OB$ (radii of circle),

$O\hat{B}A = 2x°$ (base angles of isosceles Δ)

$\therefore \quad A\hat{O}B + 2x° + 2x° = 180°$

$\Rightarrow \quad A\hat{O}B = 180° - 4x°$

(b) $A\hat{C}B = \frac{1}{2}(A\hat{O}B)$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$\Rightarrow \quad A\hat{C}B = \frac{1}{2}(180° - 4x°)$

$= 90° - 2x°$

(c) $D\hat{C}B = D\hat{C}A + A\hat{C}B$

$= x° + 90° - 2x° = 90° - x°$

$ABCD$ is a cyclic quadrilateral

$\Rightarrow \quad D\hat{A}B + D\hat{C}B = 180°$ (opp. $\angle$s of a cyclic quad. add up to 180°)

$\Rightarrow \quad D\hat{A}B + 90° - x° = 180°$

$\Rightarrow \quad D\hat{A}B = 180° - 90° + x°$

$= 90° + x°$

Alternative Solution

$D\hat{B}A = D\hat{C}A = x°$ (angles in the same segment)

$A\hat{D}B = A\hat{C}B$ (angles in the same segment)

$= 90° - 2x°$

$\therefore \quad D\hat{A}B + A\hat{D}B + D\hat{B}A = 180°$ ($\angle$ sum of a Δ)

$\Rightarrow \quad D\hat{A}B + 90° - 2x° + x° = 180°$

$\Rightarrow \quad D\hat{A}B + 90° - x° = 180°$

$\Rightarrow \quad D\hat{A}B = 180° - 90° + x°$

$= 90° + x°$

7. $x° = \dfrac{110°}{2}$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$= 55°$

$y° = 24°$ (angles in the same segment)

8. Given that, QR bisects angle PQB,

$\Rightarrow \quad P\hat{Q}R = B\hat{Q}R = x°$

Also, $Q\hat{P}R = x°$ (alternate segment theorem)

$\therefore$ ΔPQR has two equal angles.

Given that, $x° < 60°$, therefore the triangle is not an equilateral triangle.

Thus, ΔPQR is an isosceles triangle.

9. **(a)** $ABCD$ is a cyclic quadrilateral

Opposite angles of a cyclic quadrilateral are supplementary

$\therefore \quad A\hat{D}C + 131° = 180°$

$A\hat{D}C = 180° - 131°$

$= 49°.$

(b) $A\hat{O}C = 2(A\hat{D}C)$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$\Rightarrow \quad A\hat{O}C = 2(49°)$

$= 98°.$

(c) $B\hat{A}T = 20°$ (alternate segment theorem)

(d) $O\hat{A}T = 90°$ (radius $\perp$ tangent)

$\therefore \quad O\hat{A}B = 90° - B\hat{A}T$

$= 90° - 20°$

$= 70°$

10. Opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \quad 2x° + x° + 60° = 180°$

$\Rightarrow \quad 3x° = 120° \quad \Rightarrow \quad x° = 40°$

Also, $y° + 4x° - 87° = 180°$

$\Rightarrow \quad y° + (4)(40°) - 87° = 180°$

$\Rightarrow \quad y° + 160° - 87° = 180°$

$\Rightarrow \quad y° + 73° = 180° \quad \Rightarrow \quad y° = 180° - 73° = 107°$

11. $O\hat{A}D = O\hat{C}D = 90°$ (radius $\perp$ tangent)

$\therefore \quad A\hat{O}C = 180° - 44°$

$= 136°$

$x° = \dfrac{1}{2}(A\hat{O}C)$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$\Rightarrow \quad x° = \dfrac{1}{2}(136°) = 68°$

12. **(a)** $P\hat{Q}T = 50°$ (alternate segment theorem)

ΔPQT is isosceles with $PT = PQ$

$\Rightarrow \quad P\hat{T}Q = P\hat{Q}T = 50°$ (base $\angle$ of isosceles Δ)

$\therefore \quad T\hat{P}Q = 180° - 50° - 50°$

$= 180° - 100°$

$= 80°$

(b) $w = 68°$

Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle

$3x° + 2x° = 180°$

$5x° = 180°$

$x° = \dfrac{180°}{5} = 36°$

Opposite angles of a cyclic quadrilateral are supplementary

13. **(a)** $x = 55°$ because, alternate segment theorem.

(b) Tangents drawn from an external point to a circle are equal in lengths.

Therefore, $SV = SR$.

Thus ΔSVR is isosceles.

14. $x° = 38°$ (alternate segment theorem)

$A\hat{C}B = x°$ (alternate angles)

$A\hat{B}C + 60° = 180°$ (opp. $\angle$s of a cyclic quadrilateral add to 180°)

$\Rightarrow \quad A\hat{B}C = 180° - 60° = 120°$

In ΔABC,

$y° + x° + A\hat{B}C = 180°$ (angle sum of a Δ)

$\Rightarrow \quad y° + 38° + 120° = 180°$

$\Rightarrow \quad y° = 180° - 38° - 120°$

$= 22°$

15. Diameter of circle = DE

Angle subtended by diameter at any point on the circumference is 90°.

In ΔDEF, $D\hat{F}E = 180° - 82° - 8° = 90°$.

Thus DE is the diameter.

TOPIC 28

Mensuration

1. Calculate the area of a circle with radius 5.1 cm. Give your answer in terms of π

..... cm^2 [2]

[June/2018/P21/Q6] [Note: Question is modified to solve without a calculator]

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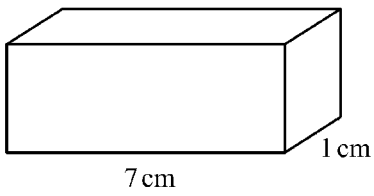
2. The diagram shows a solid cuboid with base area 7 cm^2.
The volume of this cuboid is 21 cm^3.
Work out the total surface area.



..... cm^2 [3]

[June/2018/P22/Q14]

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3. Find the volume of a cylinder of radius 5 cm and height 8 cm.
Give your answer in terms of π. Give the units of your answer.

..... [3]

[June/2018/P22/Q15] [Note: Question is modified to solve without a calculator]

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4. A water tank in the shape of a cuboid has length 1.5 metres and width 1 metre. The water in the tank is 60 centimetres deep.

Calculate the number of litres of water in the tank.

..... litres [3]

[Nov/2018/P21/Q10]

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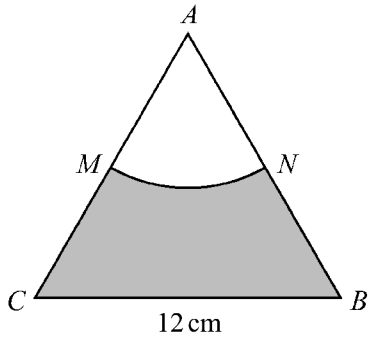
5. The diagram shows an equilateral triangle ABC with sides of length 12 cm.

AMN is a sector of a circle, centre A.
M is the mid-point of AC.

Work out the area of the shaded region. Give your answer in terms of π.



..... cm^2 [4]

[Nov/2018/P22/Q21] [Note: Question is modified to solve without a calculator]

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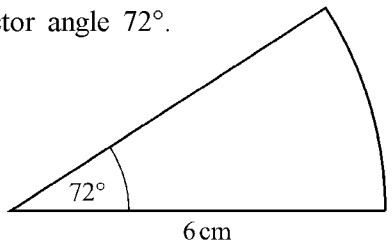
6. The diagram shows a sector of a circle with radius 6 cm and sector angle 72°.

The perimeter of this sector is $(p + q\pi)$ cm.

Find the value of p and the value of q.



p = ...

q = ... [3]

[Nov/2018/P23/Q19]

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7. The volume of a cuboid is 180 cm^3. The base is a square of side length 6 cm.
Calculate the height of this cuboid.

..... cm [2]

[June/2019/P21/Q5]

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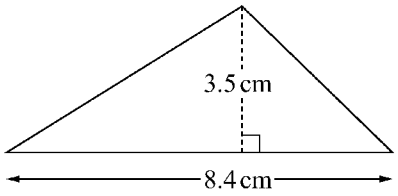
8. Calculate the area of this triangle.



..... cm^2 [2]

[June/2019/P22/Q7]

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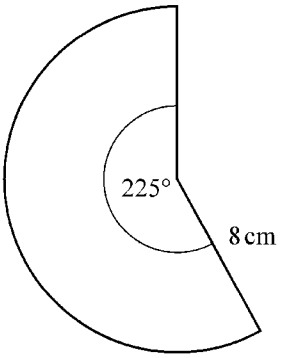
9. The diagram shows a sector of a circle with radius 8 cm and sector angle 225°.

Calculate the area of this sector. Give your answer in terms of π.



..... cm^2 [2]

[June/2019/P23/Q12] [Note: Question is modified to solve without a calculator]

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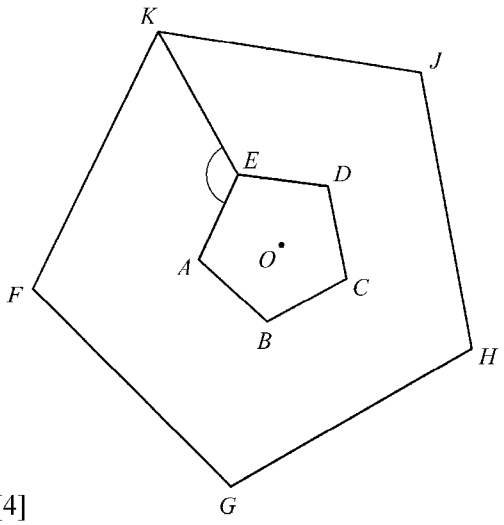
10. The diagram shows two regular pentagons.

Pentagon *FGHJK* is an enlargement of pentagon *ABCDE*, centre *O*.

(a) Find angle *AEK*.



Angle *AEK* = ... [4]

(b) The area of pentagon $FGHJK$ is 73.5 cm^2.
The area of pentagon $ABCDE$ is 6 cm^2.

Find the ratio perimeter of pentagon $FGHJK$: perimeter of pentagon $ABCDE$ in its simplest form.

..... : ..... [2]

[June/2019/P23/Q25]

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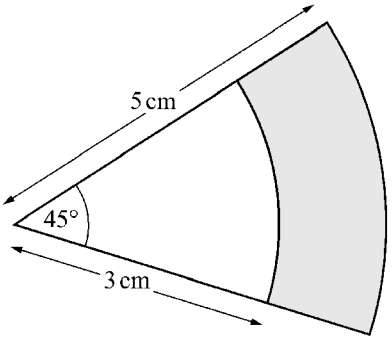
11. The diagram shows two sectors of circles with the same centre.

Calculate the shaded area. Give your answer in terms of π.



..... cm^2 [3]

[Nov/2019/P21/Q17] [Note: Question is modified to solve without a calculator]

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12. A pipe is completely full of water.
Water flows through the pipe at a speed of 1.2 m/s into a tank.
The cross-section of the pipe has an area of 6 cm^2.

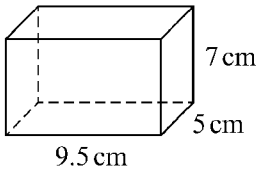
Calculate the number of litres of water flowing into the tank in 1 hour.

..... litres [4]

[Nov/2019/P21/Q22]

13. A cuboid measures 5 cm by 7 cm by 9.5 cm.

Work out the surface area of this cuboid.



..... cm^2 [3]

[Nov/2019/P23/Q9]

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14. A container is made from a cylinder and a cone, each of radius 5 cm.

The height of the cylinder is 12 cm and the height of the cone is 4.8 cm.

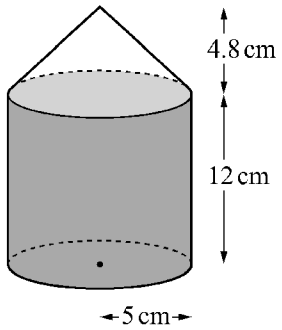
The cylinder is filled completely with water.

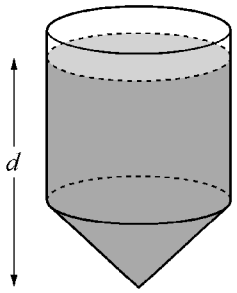
The container is turned upside down as shown below.

Calculate the depth, d, of the water.

[Do all calculations in terms of π.]

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]





d = ... cm [5]

[Nov/2019/P23/Q22] [Note: Question is modified to solve without a calculator]

ANSWERS

Topic 28 - Mensuration

1. Area of circle $= \pi(5.1)^2$

$= 26.01\pi \text{ cm}^2$

2. Let height of the cuboid be h cm.

Volume of cuboid $= 21 \text{ cm}^3$

$\Rightarrow \quad 7 \times 1 \times h = 21 \quad \Rightarrow \quad h = \dfrac{21}{7} = 3 \text{ cm}$

$\therefore$ Total surface area

$= 2(7 \times 1) + 2(1 \times 3) + 2(7 \times 3)$

$= 14 + 6 + 42 = 62 \text{ cm}^2$

3. Volume of cylinder $= \pi(5)^2(8)$

$= \pi(25)(8) = 200\pi \text{ cm}^3$

4. Depth of water $= 60$ cm

$= \dfrac{60}{100} \text{ m} = 0.6 \text{ m}$

Volume of water in the tank $= 1.5 \times 1 \times 0.6$

$= 0.9 \text{ m}^3$

$1 \text{ m}^3 = 1000$ litres

$\therefore$ Number of litres in the tank $= 0.9 \times 1000$

$= 900$ litres

5. Radius of sector, $AM = 6$ cm

ΔABC is an equilateral triangle

$\therefore \quad C\hat{A}B = 60°$

Area of shaded region

= area of ΔABC – area of sector AMN

$= \dfrac{1}{2}(12)(12)\sin 60° - \dfrac{60°}{360°} \times \pi(6)^2$

$= 72\left(\dfrac{\sqrt{3}}{2}\right) - \dfrac{1}{6} \times \pi(36)$

$= 36\sqrt{3} - 6\pi \text{ cm}^2$

6. Perimeter of sector $= 6 + 6 +$ arc length

$= 6 + 6 + \dfrac{72°}{360°} \times 2\pi(6)$

$= 12 + \dfrac{12}{5}\pi$

$\therefore \quad p = 12 \quad , \quad q = \dfrac{12}{5}$

7. Let height of cuboid be h cm

Volume of cuboid $= 6 \times 6 \times h$

$= 36h$

$\Rightarrow \quad 36h = 180$

$\Rightarrow \quad h = \dfrac{180}{36} = 5 \text{ cm}$

8. Area of $\Delta = \dfrac{1}{2}(8.4)(3.5)$

$= 14.7 \text{ cm}^2$

9. Area of sector $= \dfrac{225°}{360°} \times \pi(8)^2$

$= \dfrac{5}{8} \times \pi(64) = 40\pi \text{ cm}^2$

10. (a) One interior angle of pentagon $= \dfrac{(5-2) \times 180°}{5}$

$= \dfrac{3 \times 180°}{5}$

$= 108°$

$\therefore \quad A\hat{E}D = 108°$

Reflex $A\hat{E}D = 360° - 108°$

$= 252°$

$\therefore \quad A\hat{E}K = \dfrac{1}{2}(252°) = 126°$

(b) $\dfrac{\text{Area of } FGHJK}{\text{Area of } ABCDE} = \left(\dfrac{\text{perimeter of } FGHJK}{\text{perimeter of } ABCDE}\right)^2$

$\Rightarrow \quad \dfrac{73.5}{6} = \left(\dfrac{\text{perimeter of } FGHJK}{\text{perimeter of } ABCDE}\right)^2$

$\Rightarrow \quad \dfrac{735}{60} = \left(\dfrac{\text{perimeter of } FGHJK}{\text{perimeter of } ABCDE}\right)^2$

$\Rightarrow \quad \dfrac{49}{4} = \left(\dfrac{\text{perimeter of } FGHJK}{\text{perimeter of } ABCDE}\right)^2$

$\Rightarrow \quad \dfrac{\text{perimeter of } FGHJK}{\text{perimeter of } ABCDE} = \sqrt{\dfrac{49}{4}}$

$\Rightarrow \quad \dfrac{\text{perimeter of } FGHJK}{\text{perimeter of } ABCDE} = \dfrac{7}{2}$

$\therefore$ Required ratio = 7 : 2

11. Shaded area = Area of larger sector
− area of smaller sector

$$= \frac{45°}{360°} \times \pi(5)^2 - \frac{45°}{360°}\pi(3)^2$$
$$= \frac{1}{8} \times \pi(25) - \frac{1}{8}\pi(9)$$
$$= \frac{25}{8}\pi - \frac{9}{8}\pi$$
$$= \frac{16}{8}\pi = 2\pi \text{ cm}^2$$

12. Speed of flow of water = 1.2 m/s
$= 1.2 \times 100 = 120$ cm/s

Volume of water that flows in one second
= area of cross section × speed of flow
$= 6 \times 120 = 720 \text{ cm}^3/\text{s}$

$\therefore$ Volume of water that flows in one hour
$= 720 \times 60 \times 60$
$= 720 \times 3600$
$= 2592000 \text{ cm}^3$
$= 2592000$ ml $=$ 2592 litres

13. Surface area $= 2(9.5 \times 5) + 2(5 \times 7) + 2(7 \times 9.5)$
$= 95 + 70 + 133 = 298 \text{ cm}^2$

14. Volume of cylinder = Volume of water
$= \pi(5)^2(12)$
$= 300\pi \text{ cm}^3$

Volume of cone $= \frac{1}{3}\pi(5)^2(4.8) = 40\pi \text{ cm}^3$

When the container is turned upside down,
volume of water in cylinder $= 300\pi - 40\pi$
$= 260\pi \text{ cm}^3$

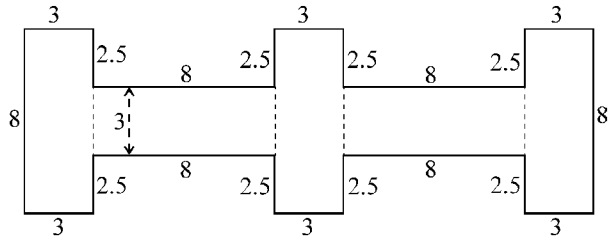
Let h cm be the depth of water in cylinder.

$\therefore \quad \pi(5)^2 h = 260\pi$

$\Rightarrow \quad h = \frac{260\pi}{25\pi} \quad \Rightarrow \quad h = \frac{52}{5} = 10.4$ cm

$\therefore$ depth, $d = 10.4 + 4.8 = 15.2$ cm

15.



Perimeter $= (6 \times 8) + (6 \times 3) + (8 \times 2.5)$
$= 48 + 18 + 20 = 86$ cm

16. Area of trapezium $= \frac{1}{2}(7 + 11)(5)$
$= \frac{1}{2}(18)(5) = 45 \text{ cm}^2$

17. Let θ be the angle of the sector

Length of arc $PQ = \frac{\theta}{360} \times 2\pi(7)$

$\Rightarrow \quad 5.5 = \frac{\theta}{360°} \times 2 \times \frac{22}{7} \times 7$

$\Rightarrow \quad 5.5 = \frac{\theta}{360°} \times 44$

$\Rightarrow \quad 5.5 = \frac{11}{90}\theta \quad \Rightarrow \quad \theta = 5.5 \times \frac{90}{11} = 45°$

Now, Area of sector $= \frac{45°}{360°} \times \frac{22}{7} \times 7^2$
$= \frac{1}{8} \times 22 \times 7$
$= \frac{77}{4} = 19.25 \text{ cm}^2$

18. Volume of cuboid $= 8 \times 4 \times 6.5$
$= 208 \text{ cm}^3$

19. 60 mm = 6 cm

$\therefore$ Area of sector $= \frac{42°}{360°} \times \pi(6)^2$
$= \frac{42°}{360°} \times \pi(36) = 4.2\pi \text{ cm}^2$

20. Total surface area of cylinder
= 2(area of circle) + curved surface area
$= 2\left(\pi(3)^2\right) + 2\pi(3)(4.5)$
$= 18\pi + 27\pi = 45\pi \text{ cm}^2$

21. Area of triangle $= 27 \text{ cm}^2$

$\Rightarrow \quad \frac{1}{2}(6)(h) = 27$

$\Rightarrow \quad 3h = 27 \quad \Rightarrow \quad h = 9$ cm

22. Total surface area of the solid
= area of circular base
+ curved surface area of cylinder
+ area of hemisphere

$= \pi(7)^2 + 2\pi(7)(12) + \frac{1}{2}\left(4\pi(7)^2\right)$
$= 49\pi + 168\pi + 98\pi$
$= 315\pi \text{ cm}^2$