REDSPOT

O LEVEL

CLASSIFIED
UNSOLVED EXAM PAPERS

MATHEMATICS
(Paper 2 - All Variants)
(Syllabus 4024)

period	2013 to 2022
contents	June & November, Paper 2 (P21 & P22) With Solutions
form	Topic By Topic
compiled for	O Levels

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TOPIC 7

Solutions of Equations

1. (a) In an athletics match Ben won the 100 m race in 9.98 s and Calvin won the 200 m race in 19.94 s.

What is the difference in their average speeds?
Give your answer in metres per second, correct to two decimal places.

Answer .... m/s [2]

(b) Two cars each complete a journey of 120 km.
The first car is driven at an average speed of x km/h.
The second car is driven at an average speed 3 km/h faster than the first car.
The first car takes 6 **minutes** longer to complete the journey.

(i) Write down an equation in x and show that it simplifies to $x^2 + 3x - 3600 = 0$.

[3]

(ii) Solve the equation $x^2 + 3x - 3600 = 0$, giving each answer correct to one decimal place.

Answer x = ... or .... [3]

(iii) How many **minutes** does the **first** car take to travel the 120 km?

Answer .... minutes [2]

[June/2013/P21/Q7]

2. (a) Solve $3(x-5)=5x-7$.

Answer $x =$ .... [2]

(b) Solve the simultaneous equations.

$$2x-y=6$$
$$4x+3y=-3$$

Answer $x =$ ....

Answer $y =$ .... [3]

[June/2013/P22/Q3(a,c)]

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3. (a) Solve $\dfrac{2}{3-x}=1$.

Answer .... [1]

(b) Factorise

(i) $5x+5y$,

Answer .... [1]

(ii) $9x^2-16$.

Answer .... [1]

(c) (i) Factorise $2x^2+5x-12$.

Answer .... [1]

(ii) Use your answer to **part (c)(i)** to solve the equation $2x^2+5x-12=0$.

Answer $x =$ .... or .... [1]

[Nov/2013/P21/Q5(a,b,c)]

4. (a) Find the value of $\dfrac{a+\sqrt{a^2+b^2}}{a^2-2ab}$ when $a=-4$ and $b=-3$.

Give your answer as a fraction.

Answer .... [2]

(b) Expand the brackets and simplify $(3x^2-1)(2x+3)-x(9x-2)$.

Answer .... [2]

(c) (i) Factorise $9x^2+5x-4$.

Answer .... [1]

(ii) Use your answer to **part (c)(i)** to solve the equation $9x^2+5x-4=0$.

Answer x = ... or .... [1]

(d) The sum of three consecutive integers is 84.

Find these three integers.

Answer .... , ... , ... [2]

[Nov/2013/P22/Q3]

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5. (a) Express as a single fraction, in its simplest form, $\dfrac{7}{p+2}-\dfrac{4}{2p-3}$.

Answer .... [3]

(b) The distance between London and York is 320 km.
A train takes x hours to travel between London and York.

(i) Write down an expression, in terms of x, for the average speed of the train.

Answer .... km/h [1]

(ii) A car takes $2\frac{1}{2}$ hours longer than a train to travel between London and York.
The average speed of the train is 80 km/h greater than the average speed of the car.

Form an equation in x and show that it simplifies to $2x^2 + 5x - 20 = 0$.

[3]

(iii) Solve the equation $2x^2 + 5x - 20 = 0$, giving your answers correct to 2 decimal places.

Answer $x =$ .... or .... [3]

(iv) Hence find the average speed of the car correct to the nearest km/h.

Answer .... km/h [2]

[Nov/2013/P22/Q11]

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6. (a) Solve the simultaneous equations.

$$2x - 3y = 14$$
$$6x + 4y = 3$$

Answer $x =$ ....

$y =$ .... [3]

(b) Solve $x(x-4) = 6 + x$.

Answer $x =$ .... or .... [3]

[June/2014/P21/Q1(b,c)]

ANSWERS

Topic 7 - Solutions of Equations

1. **(a)** Speed diff. $= \dfrac{200}{19.94} - \dfrac{100}{9.98} = 0.01$ m/s

(b) **(i)** Time taken by 1st car $= \dfrac{120}{x}$ h

Time taken by 2nd car $= \dfrac{120}{x+3}$ h

$\therefore \quad \dfrac{120}{x} - \dfrac{120}{x+3} = \dfrac{6}{60}$

$\Rightarrow 120\left(\dfrac{x+3-x}{x(x+3)}\right) = \dfrac{1}{10}$

$\Rightarrow x^2 + 3x - 3600 = 0$

(ii) $x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(1)(-3600)}}{2(1)}$

$= \dfrac{-3 \pm \sqrt{14409}}{2}$

$\therefore \; x = 58.5$ or -61.5

(iii) Time taken by 1st car $= \dfrac{120}{58.5}$h

$= 123$ minutes.

2. **(a)** $3(x-5) = 5x - 7$

$\Rightarrow 3x - 15 = 5x - 7 \quad \Rightarrow \quad x = -4.$

(b) $2x - y = 6$ ....(1), $\quad 4x + 3y = -3$ ....(2)

From (1): $\quad y = 2x - 6$ ....(3)

Substitute (3) into (2) to obtain, $x = 1.5$

Substitute $x = 1.5$ into (3) to get, $y = -3$

3. **(a)** $\dfrac{2}{3-x} = 1 \quad \Rightarrow \quad 2 = 3 - x \quad \Rightarrow \quad x = 1$

(b) **(i)** $5(x+y)$

(ii) $9x^2 - 16 = (3x-4)(3x+4)$

(c) **(i)** $2x^2 + 5x - 12 = (2x-3)(x+4)$

(ii) Using (c)(i), $(2x-3)(x+4) = 0$

$\therefore \; x = \dfrac{3}{2}$ or -4

4. **(a)** $\dfrac{-4 + \sqrt{(-4)^2 + (-3)^2}}{(-4)^2 - 2(-4)(-3)}$

$= \dfrac{-4 + \sqrt{16+9}}{16-24} = \dfrac{-4+\sqrt{25}}{-8} = -\dfrac{1}{8}$

(b) $(3x^2 - 1)(2x+3) - x(9x-2)$

$= 6x^3 + 9x^2 - 2x - 3 - 9x^2 + 2x$

$= 6x^3 - 3 = 3(2x^3 - 1)$

(c) **(i)** $9x^2 + 5x - 4 = (x+1)(9x-4)$

(ii) From (c) (i), $\quad (x+1)(9x-4) = 0$

$\therefore \quad x = -1$ or $\dfrac{4}{9}$

(d) Let the integers be $x,\; x+1,\; x+2$

$x + (x+1) + (x+2) = 84$

$\Rightarrow 3x + 3 = 84 \quad \Rightarrow \quad x = 27$

$\therefore$ the integers are 27, 28, 29

5. **(a)** $\dfrac{7}{p+2} - \dfrac{4}{2p-3}$

$= \dfrac{7(2p-3) - 4(p+2)}{(p+2)(2p-3)} = \dfrac{10p-29}{(p+2)(2p-3)}$

(b) **(i)** Average speed $= \dfrac{320}{x}$ km/h.

(ii) Car av. speed $= \dfrac{320}{x+\frac{5}{2}} = \dfrac{640}{2x+5}$ km/h

Now, $\dfrac{320}{x} - \dfrac{640}{2x+5} = 80$

$\Rightarrow 320\left(\dfrac{1}{x} - \dfrac{2}{2x+5}\right) = 80$

$\Rightarrow \dfrac{5}{x(2x+5)} = \dfrac{1}{4}$

$\Rightarrow 2x^2 + 5x - 20 = 0$

(iii) $2x^2 + 5x - 20 = 0$. By quad. formula,

$x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(2)(-20)}}{2(2)}$

$= \dfrac{-5 \pm \sqrt{185}}{4}$

$\therefore \; x = 2.15$ or -4.65

TOPIC 12

Graphs of Functions

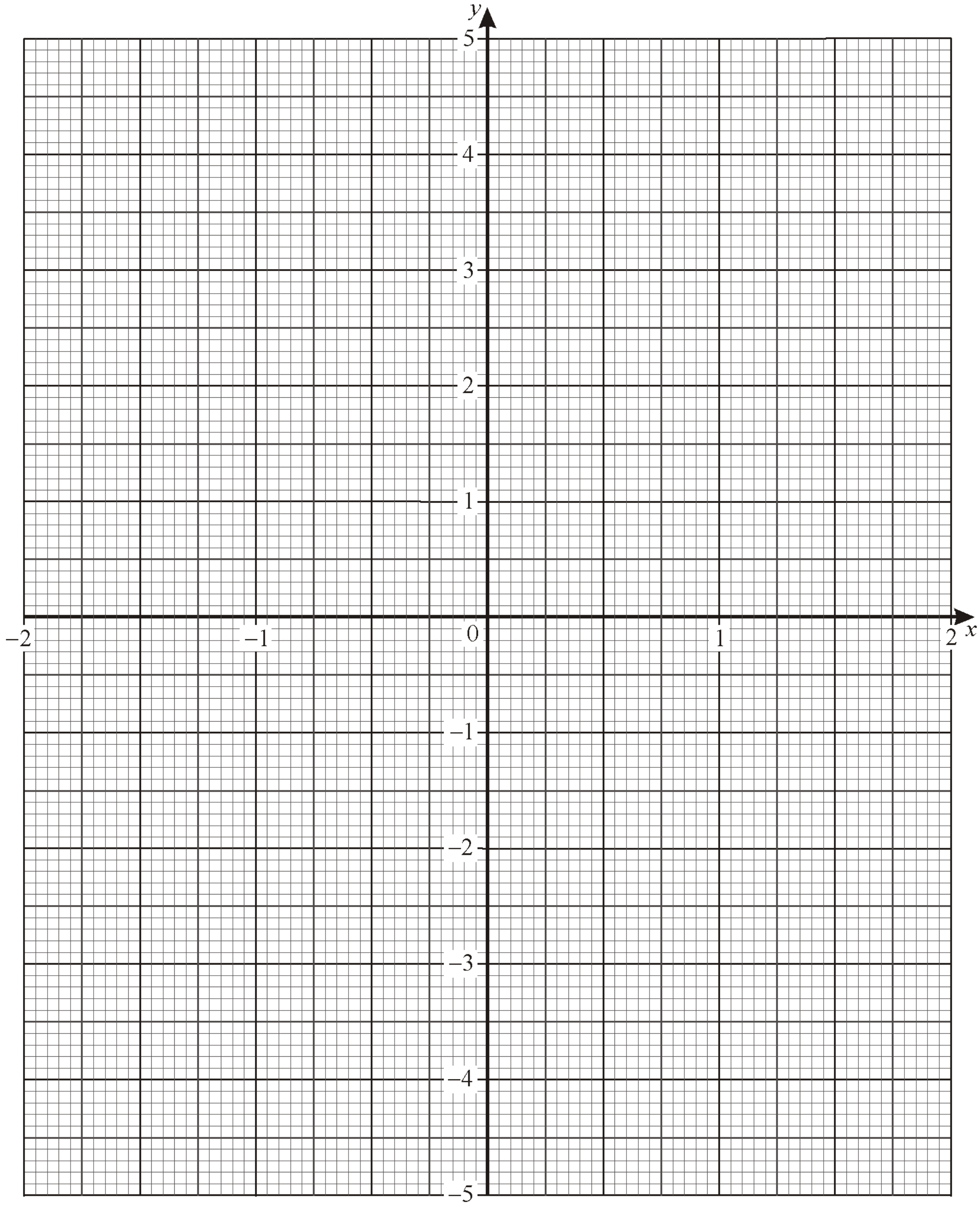
1. The variables x and y are connected by the equation $y = x + \dfrac{1}{x}$.

The table below shows some values of x and the corresponding values of y.

The values of y are correct to 2 decimal places where appropriate.

x	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	4.25	2.5	2.08	2	2.05	2.17	2.32	2.5

(a) On the grid, plot the points given in the table and join them with a smooth curve. [2]



(b) By drawing a tangent, estimate the gradient of the curve when $x = 0.75$.

Answer .... [2]

(c) Let $f(x) = x + \frac{1}{x}$.

(i) Given that $f(a) = b$, find $f(-a)$ in terms of b.

Answer .... [1]

(ii) Hence, or otherwise, complete the table below for $y = x + \frac{1}{x}$.

x	-2	-1.75	-1.5	-1.25	-1	-0.75	-0.5	-0.25
y					-2			

[1]

(iii) On the grid, draw the graph of $y = x + \frac{1}{x}$ for $-2 \le x \le -0.25$. [1]

(iv) Write down an estimate for the gradient of the curve when $x = -0.75$.

Answer .... [1]

(d) (i) On the grid, draw the graph of the straight line $y = 4 - x$. [1]

(ii) Write down the x-coordinate of each of the points where the graphs of $y = 4 - x$ and $y = x + \frac{1}{x}$ intersect.

Answer x = ... and .... [1]

(iii) Find the equation for which these x values are the solutions.

Give your equation in the form $Ax^2 + Bx + C = 0$.

Answer .... [2]

[Nov/2013/P22/Q9]

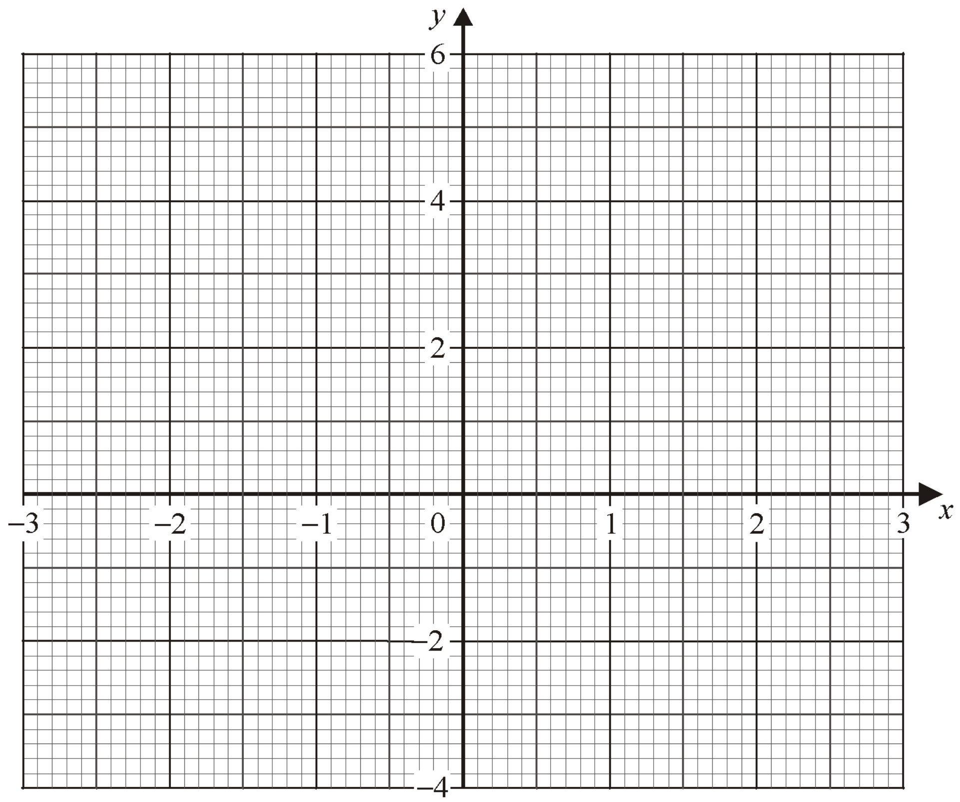
2. The variables x and y are connected by the equation $y = \frac{x^3}{2} - 3x + 1$.

Some corresponding values of x and y are given in the table below.

x	-3	-2	-1	0	1	2	3
y		3	3.5	1	-1.5	-1	

(a) Complete the table. [2]

(b) On the grid below, plot the points from the table and join them with a smooth curve.



[2]

(c) Use your graph to solve the equation $\frac{x^3}{2} - 3x + 1 = 0$.

Answer .... [2]

(d) By drawing a tangent, find the gradient of the curve at the point (–2, 3).

Answer .... [2]

(e) The line *AB* intersects the curve at point *P*.
The coordinates of point *A* are (0, 5). The coordinates of point *B* are (2, –3).

(i) Find the equation of line *AB*.

Answer .... [2]

(ii) The *x*-coordinate of point *P* is a solution of the equation $\frac{x^3}{2} + Cx + D = 0$.

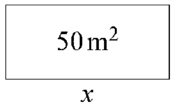
Find *C* and *D*.

Answer C = ...

D = ... [2]

[June/2014/P21/Q7]

3. Adil wants to fence off some land as an enclosure for his chickens.
The enclosure will be a rectangle with an area of 50 m^2.



(a) The enclosure is x m long.

Show that the total length of fencing, L m, required for the enclosure is given by

$$L = 2x + \frac{100}{x}.$$

[2]

(b) The table below shows some values of x and the corresponding values of L, correct to one decimal place where appropriate, for $L = 2x + \frac{100}{x}$.

x	2	4	6	8	10	12	14	16	18	20
L	54	33	28.7	28.5	30	32.3	35.1	38.3		

Complete the table. [2]

(c) On the grid, next page,
draw a horizontal x-axis for $0 \le x \le 20$ using a scale of 1 cm to represent 2 m
and a vertical L-axis for $0 \le L \le 60$ using a scale of 2 cm to represent 10 m.

On the grid, plot the points given in the table and join them with a smooth curve. [3]

(d) Adil only has 40 m of fencing.

Use your graph to find the range of values of x that he can choose.

Answer .... $\le x \le$ .... [2]

(e) (i) Find the minimum length of fencing Adil could use for the enclosure.

Answer .... m [1]

(ii) Find the length and width of the enclosure using this minimum length of fencing.
Give your answers correct to the nearest metre.

Answer Length = ... m Width = ... m [1]

(f) Suggest a suitable length and width for an enclosure of area 100 m^2, that uses the minimum possible length of fencing.

Answer Length = ... m Width = ... m [1]

ANSWERS

Topic 12 - Graphs of Functions

1. **(a)** Refer to graph

(b) Using (1.5, 1.5) and (0.1, 2.6), Gradient $= \dfrac{2.6-1.5}{0.1-1.5} = -0.786$

(c) **(i)** $f(a) = a + \dfrac{1}{a} \Rightarrow b = a + \dfrac{1}{a}$

$f(-a) = -a - \dfrac{1}{a} = -b, \quad \therefore f(-a) = -b$

(ii)

x	−2	−1.75	−1.5	−1.25	−1	−0.75	−0.5	−0.25
y	−2.5	−2.32	−2.17	−2.05	−2	−2.08	−2.5	−4.25

(iii) Refer to graph

(iv) Tangent at $x = -0.75$ would be parallel to the tangent drawn in (b), so gradient $= -0.786$

(d) **(i)** Refer to graph

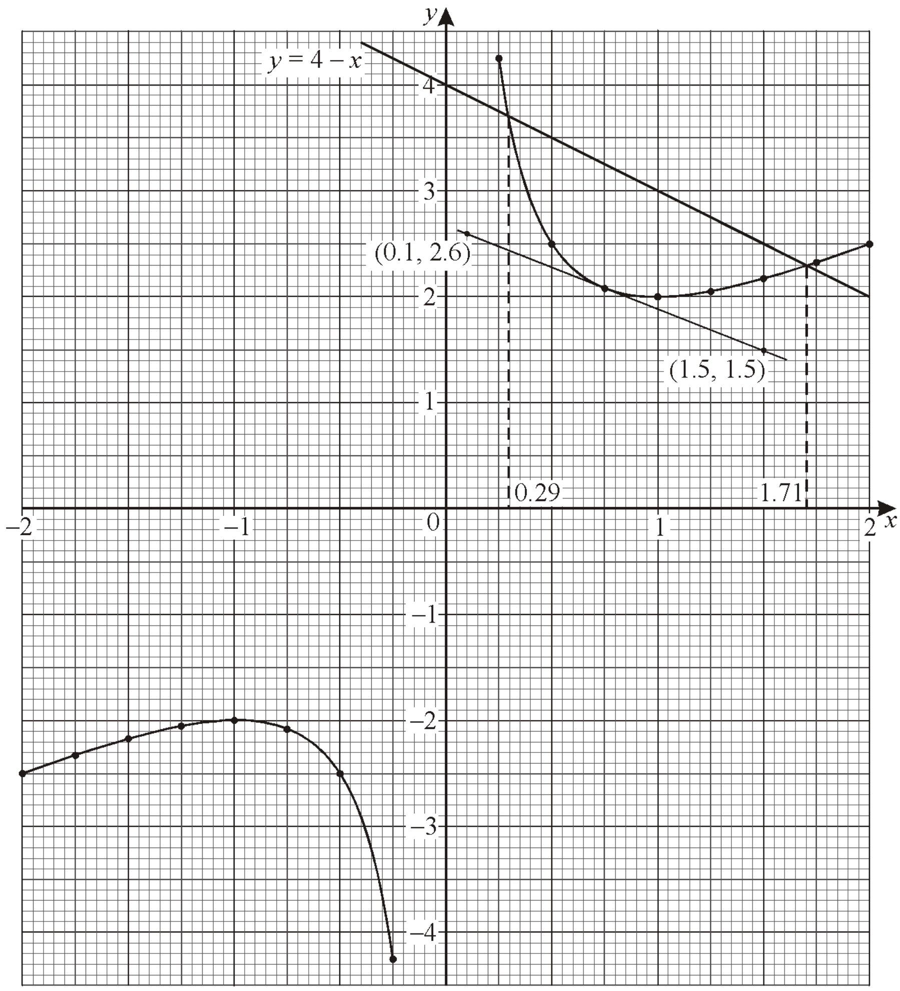
(ii) $x = 1.29$ and 1.71

(iii) Solving line and curve simultaneously,

$x + \dfrac{1}{x} = 4 - x$

$\Rightarrow x^2 + 1 = 4x - x^2$

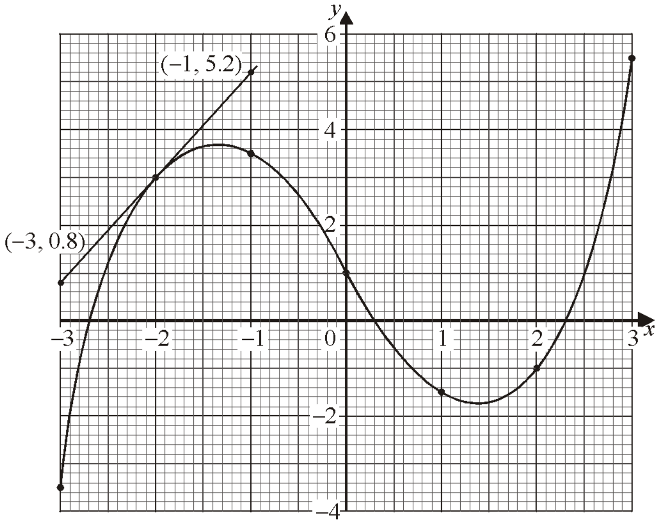
$\Rightarrow 2x^2 - 4x + 1 = 0$



2. **(a)** For $x = -3$, $y = \frac{(-3)^3}{2} - 3(-3) + 1 = -3.5$

For $x = 3$, $y = \frac{3^3}{2} - 3(3) + 1. = 5.5$

(b)



(c) $\frac{x^3}{2} - 3x + 1 = 0 \Rightarrow y = 0$

From graph, $x = -2.7,\ 0.3,\ 2.3$

(d) Taking points $(-3, 0.8)$ and $(-1, 5.2)$,

Gradient $= \frac{5.2 - 0.8}{-1 + 3} = 2.2$

(e) **(i)** Grad. of $AB = \frac{-3-5}{2-0} = -4$

Equation is, $y = -4x + 5$

(ii) Solving simult. line AB and curve,

$\frac{x^3}{2} - 3x + 1 = -4x + 5$

$\Rightarrow \frac{x^3}{2} + x - 4 = 0, \quad \therefore C = 1, \quad D = -4$

3. **(a)** Rectangle width, $w = \frac{50}{x}$

Total length, $L = 2x + 2w$

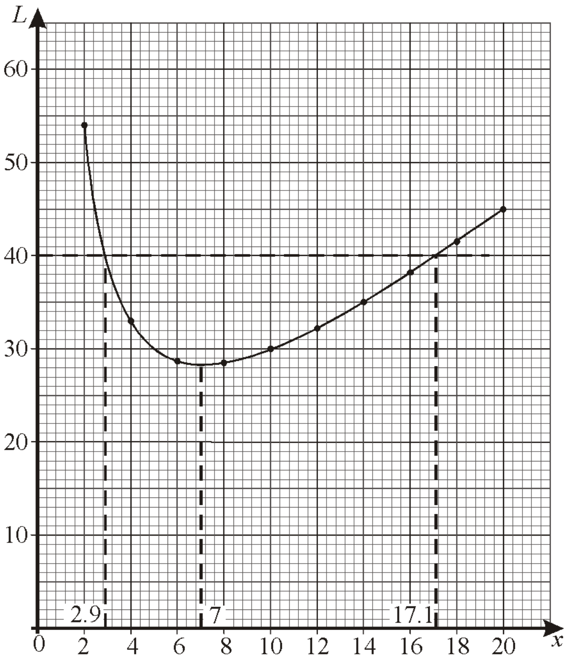
$\Rightarrow L = 2x + 2\left(\frac{50}{x}\right)$

$\Rightarrow L = 2x + \frac{100}{x}$

(b) When $x = 18$, $L = 2(18) + \frac{100}{18} = 41.56$

When $x = 20$, $L = 2(20) + \frac{100}{20} = 45$

(c)



(d) From graph, $2.9 \le x \le 17.1$

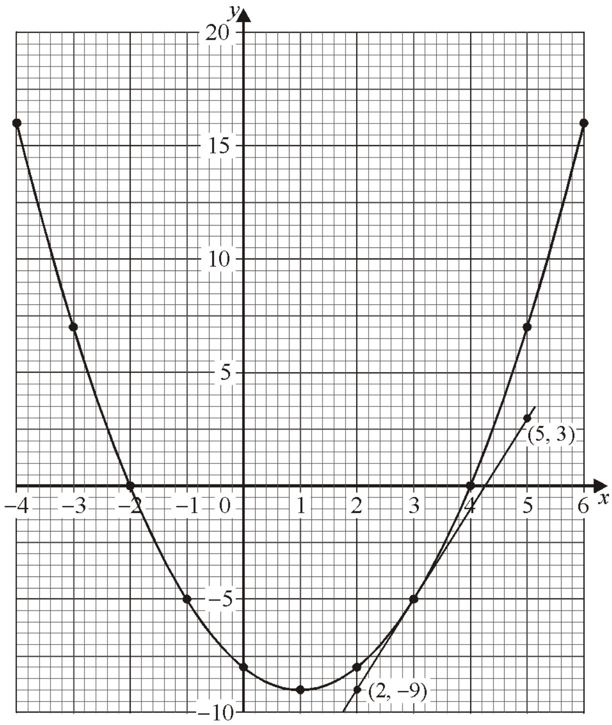
(e) **(i)** From graph, min. length = 28.2 m.

(ii) Length = 7 m. width $= \frac{50}{7} \approx 7$ m

(f) For min. perimeter, the rectangle must be a square. So, length = 10 m, width = 10 m

4. **(a)** **(i)** For $x = -4$, $y = (-4)^2 - 2(-4) - 8 = 16$

For $x = 6$, $y = (6)^2 - 2(6) - 8 = 16$

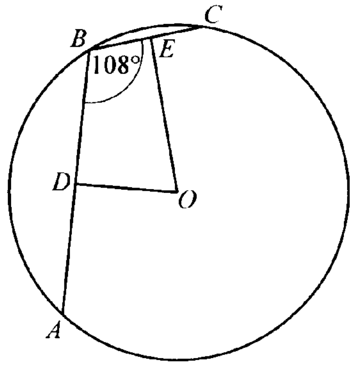


TOPIC 19

Circle Properties

1. (a) AB and BC are chords of a circle centre O.
D is the midpoint of AB and E is the midpoint of BC.
$A\hat{B}C = 108°$.

Find $D\hat{O}E$ giving your reasons.

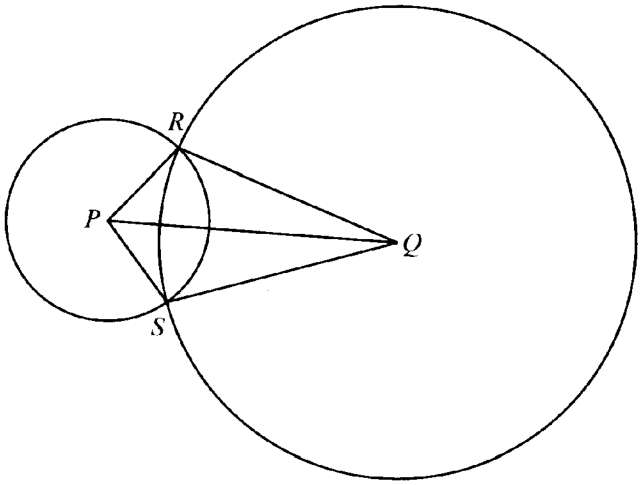


Answer $D\hat{O}E =$ .... because ....

.... [2]

(b) A circle centre P and a circle centre Q intersect at R and S.

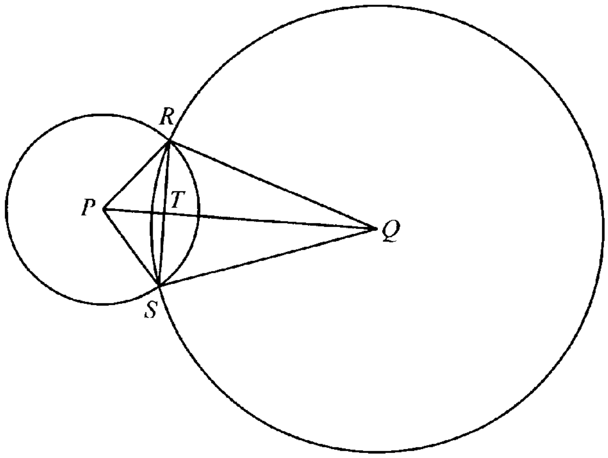
(i) Show that triangle PRQ is congruent to triangle PSQ.



[3]

(ii) RS and PQ intersect at T.

(a) State the name of the special quadrilateral $PRQS$.



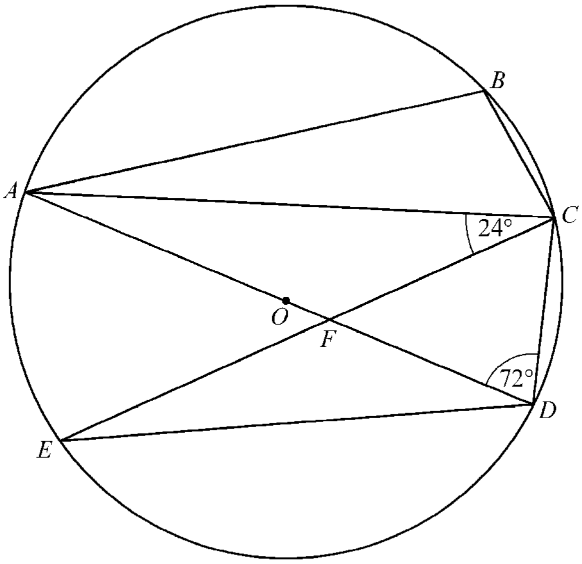
Answer .... [1]

(b) Find $P\hat{T}R$

Answer .... [1]

[Nov/2013/P22/Q4]

2. A, B, C, D and E are points on a circle with centre O.
AD is a diameter of the circle and F is the point of intersection of AD and CE.
$A\hat{C}E = 24°$ and $A\hat{D}C = 72°$.



(a) Find

(i) $A\hat{D}E$,

Answer .... [1]

(ii) $C\hat{E}D$,

Answer .... [1]

(iii) $C\hat{F}D$,

Answer .... [1]

(iv) $A\hat{B}C$.

Answer .... [1]

(b) Given that $DC = 4.5$ cm, calculate

(i) the diameter of the circle,

Answer .... cm [2]

(ii) DE.

Answer .... cm [2]

[June/2014/P21/Q4]

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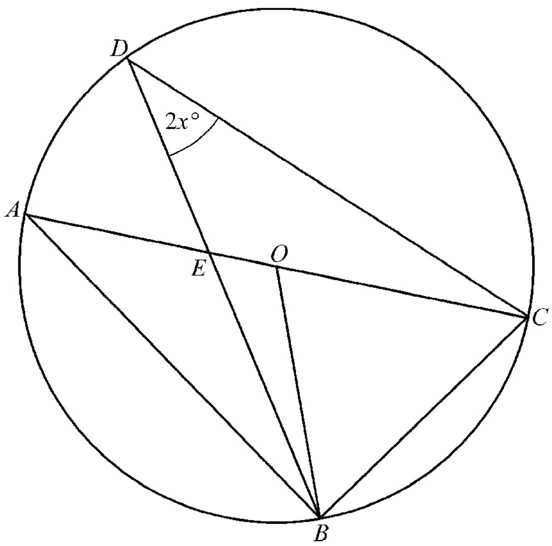
3. A, B, C and D are points on the circumference of a circle, centre O.
The diameter AC intersects BD at E.
$B\widehat{D}C = 2x°$.



(i) Find an expression, in terms of x, for

(a) $B\widehat{A}C$,

Answer .... [1]

(b) $B\widehat{O}C$,

Answer .... [1]

(c) $O\widehat{C}B$.

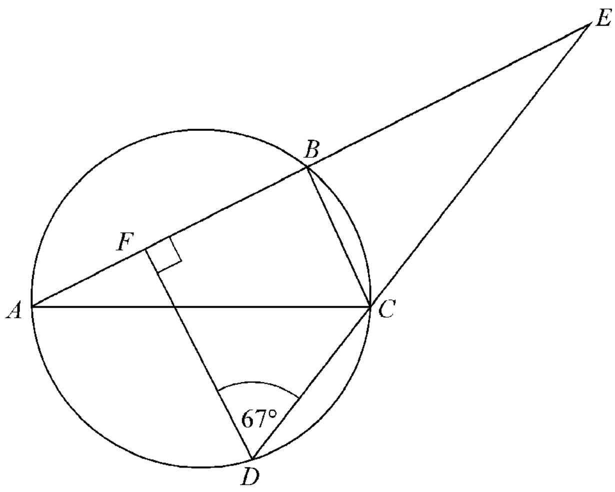
Answer .... [1]

(ii) Calculate x when $O\widehat{B}E = x°$ and $D\widehat{E}C = 123°$.

Answer $x =$ .... [3]

[Nov/2014/P21/Q10(a)]

4. A, B, C and D are points on the circumference of the circle and AC is a diameter.
$AFBE$ and DCE are straight lines.
DF is perpendicular to AE and $C\widehat{D}F = 67°$.



(i) Find $A\widehat{E}D$.

Answer $A\widehat{E}D =$ .... [1]

(ii) Find $C\widehat{B}E$, giving a reason for your answer.

Answer $C\widehat{B}E =$ .... because ....

.... [1]

(iii) Explain why DF is parallel to CB.

Answer ....

.... [1]

[Nov/2014/P22/Q2(a)]

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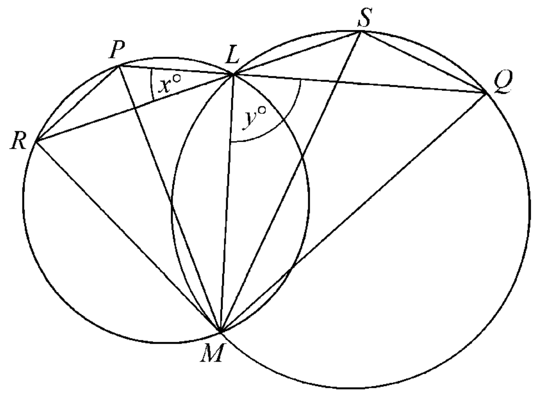
5. Two circles intersect at L and M.
R and P are on the circumference of one circle. S and Q are on the circumference of the other circle.
PLQ and RLS are straight lines.
$P\hat{L}R = x°$ and $M\hat{L}Q = y°$.



(i) Complete the proof that $S\widehat{M}Q = x°$.

<u>Statement</u> <u>Reason</u>

$x° = P\hat{L}R = S\hat{L}Q$ ....

$S\hat{L}Q = S\widehat{M}Q = x°$ .... [2]

ANSWERS

Topic 19 - Circle Properties

1. **(a)** $D\hat{O}E = 180° - 108° = 72°$

Because, D and E are midpoints of AB and BC. The line from the center bisects the chord at 90°.

(b) **(i)** $PR = PS$ (radii of circle)

$QR = QS$ (radii of circle)

PQ is common to both triangles

$\therefore \quad \Delta PRQ \equiv \Delta PSQ$ (SSS)

(ii) **(a)** PRQS is a Kite.

(b) $P\hat{T}R = 90°$

2. **(a)** **(i)** $A\hat{D}E = 24°$ ($\angle$s in same segment)

(ii) ACD is a right triangle, $A\hat{C}D = 90°$

$C\hat{A}D = 90° - 72° = 18°$

$\therefore\ C\hat{E}D = C\hat{A}D = 18°$

(iii) $C\hat{F}D$ is exterior angle for ΔDEF,

$\therefore\ C\hat{F}D = 24° + 18° = 42°$

(iv) $ABCD$ is a cyclic quadrilateral

$\therefore \quad A\hat{B}C = 180° - 72° = 108°$

(b) **(i)** ACD is a right angled triangle.

$$\cos 72° = \frac{4.5}{AD} \quad \Rightarrow \quad AD = 14.56 \text{ cm}$$

(ii) Using sine rule on ΔECD,

$$\frac{DE}{\sin 66°} = \frac{4.5}{\sin 18°} \quad \Rightarrow \quad DE = 13.3 \text{ cm}$$

3. **(i)** **(a)** $B\hat{A}C = 2x°$ ($\angle$s in same segment)

(b) $B\hat{O}C = 2(2x°) = 4x°$

($\angle$at centre is $2 \times \angle$at circumference).

(c) $A\hat{B}C = 90°$ ($\angle$ in semi-circle)

$\therefore \quad O\hat{C}B = 90° - 2x°$

(ii) ΔOAB is an isosceles triangle.

$A\hat{B}E = 2x° - x° = x°$

$\Rightarrow \quad A\hat{C}D = x°$ ($\angle$s in same segment)

In ΔCDE,

$2x° + x° + 123° = 180° \quad \Rightarrow \quad x° = 19°$

4. **(i)** Consider right angled $\Delta F\hat{E}D$

$F\hat{E}D = 90° - 67° = 23°$

$\therefore \quad A\hat{E}D = 23°$

(ii) $C\hat{B}E = 90°$ because $A\hat{B}C = 90°$.

Angle ABC is subtended by diameter AC.

(iii) $D\hat{F}B = C\hat{B}E = 90°$

Since the two corresponding angles are equal, therefore DF and CB are parallel.

5. **(i)** $x° = P\hat{L}R = S\hat{L}Q$ (vertically opp. angles)

$S\hat{L}Q = S\hat{M}Q = x°$ ($\angle$s in same segment)

(ii) $M\hat{L}P = 180° - y°$ ($\angle$s on a straight line)

$P\hat{R}M = 180° - (180° - y°) = y°$

$\therefore\ P\hat{R}M = y°$

Reason: $PRML$ is a cyclic quadrilateral. So, opposite angles are supplementary.

(iii) Triangles PRM and QSM are similar.

Reason: $R\hat{M}P = S\hat{M}Q = x°$

$P\hat{R}M = Q\hat{S}M = y°$

6. **(i)** $E\hat{B}C = E\hat{A}C = 72°$ ($\angle$s in same segment)

(ii) In ΔAXE,

$A\hat{X}E = 180° - 25° - 72°$ ($\angle$ sum of a Δ)

$= 83°$

$\therefore\ C\hat{X}B = A\hat{X}E = 83°$

(iii) $ACDE$ is a cyclic quadrilateral,

$\therefore \quad E\hat{D}C = 180° - 72° = 108°$.