REDSPOT Fresh Minds. New Thoughts

About O Level

MATHEMATICS
(TOPICAL)

About Thinking Process

In solving mathematical problems, we always work backward. After indentifying our main target, we go 'backward' to look for the 'easier' targets until we are able to solve the problems.

Thinking process reveals how the teacher actually goes about solving a sum in the above-said manner.

About Teacher's Comments

It reveals the extra but relevant information which is not required as part of the solutions but are extremely useful in knowing how the solutions are arrived.

period	**2011 to 2022**
contents	**June & November, Paper 1 & 2, Worked Solutions**
form	**Year By Year**
compiled for	**O Level**
special features	**Thinking Process, Teacher's Comments**

Appointed Agents & Wholesalers in PAKISTAN:

BOOK LAND
Urdu Bazaar, Lahore. Tel: 042-37124656

NATIONAL BOOK SERVICE
Urdu Bazaar, Lahore. Tel: 042-37247310.

LAROSH BOOKS
Urdu Bazaar Lahore. Tel: 042-37312126.

BURHANI BOOK CENTRE
New Urdu Bazar, Karachi, Tel: 021-32634718

MARYAM ACADEMY
Urdu Bazaar, Karachi, Tel: 0331-2425264

TARIQ BOOK TOWN
Samar Garden, Hydari North nazimzbad, Karachi
Tel: 021-34555918, 36649425

REHMAN BOOKS
College Road, Rawalpindi
Tel: 051-5770603, 0313-5618976

WELCOME BOOKS
Soneri Masjid Road, Peshawar Cantt.
Tel: 091-5274932, 0300-5860068

Tel No: 042-35201010
Mobile No: 0300-8447654
E-Mail: info@redspot.com.pk
Website: www.redspot.com.pk
Address: P.O. Box 5041, Model Town, Lahore, Pakistan.

CONTENTS

Revised Syllabus

Topic 1	Numbers
Topic 1a	Everyday Mathematics
Topic 2	Indices and Standard Form
Topic 3	Inequalities
Topic 4	Algebraic Expressions and Manipulations
Topic 4a	Variations
Topic 5	Solutions of Equations and Simultaneous Equations
Topic 6	Co-ordinate Geometry
Topic 7	Graphs of Functions and Graphical Solutions
Topic 8	Graphs in Practical Situations and Travel Graphs
Topic 9	Similarity and Congruency
Topic 10	Mensuration
Topic 11	Symmetry
Topic 12	Loci and Geometrical Constructions
Topic 13	Angles and Circle Properties
Topic 14	Trigonometry and Bearings
Topic 15	Probability
Topic 16	Transformation
Topic 17	Vectors in Two Dimensions
Topic 18	Statistics
Topic 19	Sets and Venn Diagrams
Topic 20	Matrices
Topic 21	Functions
Topic 22	Problem-Solving and Patterns

Revision

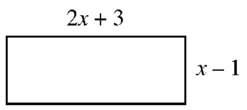
June **2022** Paper 1 & 2
December **2022** Paper 1 & 2

Topic 10

Mensuration

1 *(J2011/P1/Q26)*

The diagram shows a rectangle with length $(2x+3)$ cm and width $(x-1)$ cm .



(a) The area of the rectangle is 12 cm^2.
Form an equation in x and show that it reduces to $2x^2 + x - 15 = 0$. [2]

(b) Solve $2x^2 + x - 15 = 0$. [2]

(c) Find the perimeter of the rectangle. [1]

Thinking Process

(a) ✎ Use area = length × width.

(b) ✎ Solve by grouping.

(c) To find the perimeter ✎ substitute the value of x found in (b) into length and width of the rectangle.

Solution

(a) Area of rectangle $= l \times w$

$\Rightarrow \quad 12 = (2x+3)(x-1)$

$\Rightarrow \quad 12 = 2x^2 + 3x - 2x - 3$

$\Rightarrow \quad 2x^2 + x - 15 = 0$ **Shown.**

(b)

$$2x^2 + x - 15 = 0$$
$$2x^2 + 6x - 5x - 15 = 0$$
$$2x(x+3) - 5(x+3) = 0$$
$$(x+3)(2x-5) = 0$$

$\Rightarrow \quad x = -3$ or $x = \frac{5}{2}$

$\therefore \quad x = -3$ or 2.5 **Ans.**

(c) Using $x = 2.5$ from part (b), we have,

length $= 2(2.5) + 3 = 8$ cm

width $= 2.5 - 1 = 1.5$ cm

$\therefore$ perimeter $= 2(l + w)$

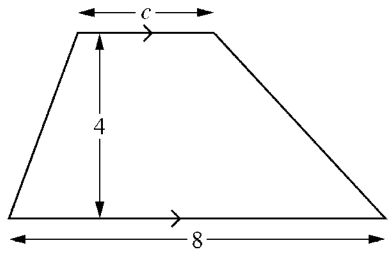
$= 2(8 + 1.5) = 19$ cm **Ans.**

2 *(J2011/P2/Q2)*

(a) The formula for the area of a trapezium is $A = \frac{1}{2}h(c+d)$.

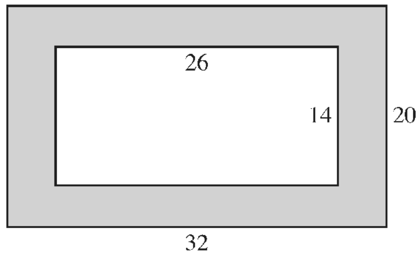
(i) Find an expression for c in terms of A, h and d. [2]

(ii)



The diagram shows a trapezium with dimensions given in centimetres.
The perpendicular distance between the parallel lines is 4 cm.
The area of the trapezium is 22 cm^2.
Find c. [1]

(b)



In the diagram, the shaded area represents a rectangular picture frame.
The outer rectangle is 32 cm by 20 cm.
The inner rectangle is 26 cm by 14 cm.
All measurements are given to the nearest centimetre.

(i) Calculate the lower bound of the perimeter of the outer rectangle. [2]

(ii) Calculate the upper bound of the area of the frame. [3]

Thinking Process

(a) (i) Make c the subject of formula.

(ii) To find c ✎ use the formula for the area of a trapezium.

(b) (i) To find the lower bound of the perimeter ✎ find the lowest possible length and width ✎ subtract 0.5 cm from each of the length and width of the rectangle.

(ii) To find the greatest area of the frame ✎ add 0.5 cm to the length and width of the outer rectangle, and subtract 0.5 from length and width of the inner rectangle.

Solution

(a) (i)

$$A = \frac{1}{2}h\,(c+d)$$
$$2A = h\,(c+d)$$
$$\frac{2A}{h} = c + d$$
$$c = \frac{2A}{h} - d \quad \textbf{Ans.}$$

(ii) From (a) (i), $c = \frac{2A}{h} - d$

$\Rightarrow \quad c = \frac{2(22)}{4} - 8$

$= 11 - 8$

$= 3$ cm **Ans.**

(b) (i) For outer rectangle,

least possible length $= 32 - 0.5 = 31.5$ cm

least possible width $= 20 - 0.5 = 19.5$ cm

lower bound of the perimeter $= 2(31.5 + 19.5)$

$= 2(51)$

$= 102$ cm **Ans.**

(ii) Upper bound of the area of the outer rectangle $= (32 + 0.5) \times (20 + 0.5)$

$= 32.5 \times 20.5 = 666.25$ cm^2

Lower bound of the area of the inner rectangle $= (26 - 0.5) \times (14 - 0.5)$

$= 25.5 \times 13.5 = 344.25$ cm^2

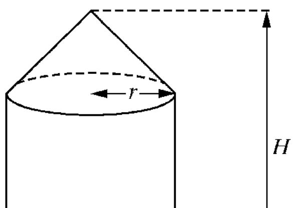
Upper bound of the area of the frame

$= 666.25 - 344.25 = 322$ cm^2 **Ans.**

> Note that:
> Upper bound or greatest area of the frame = greatest area of the outer rectangle – least area of the inner rectangle.

3 *(J2011/P2/Q11)*

[Volume of a cone $= \frac{1}{3}\pi r^2 h$]



The solid above consists of a cone with base radius r centimetres on top of a cylinder of radius r centimetres. The height of the cylinder is twice the height of the cone. The total height of the solid is H centimetres.

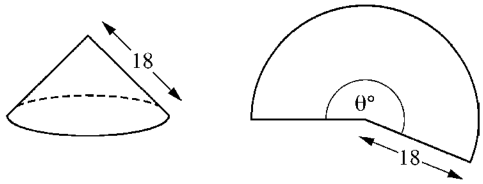
(a) Find an expression, in terms of π, r and H, for the volume of the solid.
Give your answer in its simplest form. [3]

(b) It is given that $r = 10$ and the height of the **cone** is 15 cm.

(i) Show that the slant height of the cone is 18.0 cm, correct to one decimal place. [2]

(ii) Find the circumference of the base of the cone. [2]

(iii) The curved surface area of the cone can be made into the shape of a sector of a circle with angle $\theta°$. Show that θ is 200, correct to the nearest integer.



[2]

(iv) Hence, or otherwise, find the **total** surface area of the solid. [3]

Thinking Process

(a) Volume of the solid = volume of cylinder + volume of cone.

(b) (i) Apply pythagoras theorem.

(ii) Circumference $= 2\pi r$.

(iii) Circumference of the base of cone = arc length of the sector.

(iv) To find the total surface area ✐ find the areas of the circular base, the curved surface of cylinder and the curved suface area of the cone.

Solution

(a) Let height of the cone $= h$ cm.

$\Rightarrow$ height of cylinder $= 2h$ cm

$\therefore \ h + 2h = H \ \Rightarrow \ h = \frac{1}{3}H$

Volume of solid

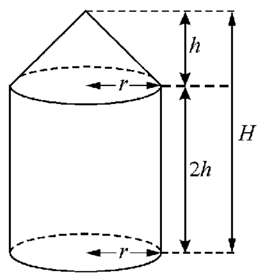
$= \pi r^2(2h) + \frac{1}{3}\pi r^2 h$

$= 2\pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \frac{7}{3}\pi r^2 h$

$= \frac{7}{3}\pi r^2 (\frac{1}{3}H)$

$= \frac{7}{9}\pi r^2 H$ cm^3 **Ans.**



(b) (i) Applying pythagoras theorem,

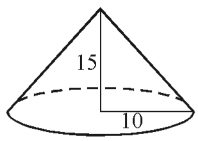
slant height

$= \sqrt{(10)^2 + (15)^2}$

$= \sqrt{325}$

$= 18.02$

≈ 18.0 cm **Shown.**



(ii) Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times 10$

$= 62.8574$

≈ 62.9 cm **Ans.**

(iii) Arc length of sector = circumference of the base of cone

$$\frac{\theta}{360} \times 2\pi(18) = 2\pi(10)$$

$$\frac{\theta}{10} = 20$$

$$\theta = 200° \quad \textbf{Shown.}$$

(iv) Total surface area

$= \pi r^2 + 2\pi rh + \pi rl$

$= \pi(10)^2 + 2\pi(10)(30) + \pi(10)(18)$

$= 100\pi + 600\pi + 180\pi$

$= 880\pi$

$= 2764.96 \approx 2765 \text{ cm}^2$ **Ans.**

4 *(N2011/P2/Q3)*

(a)

Diagram I

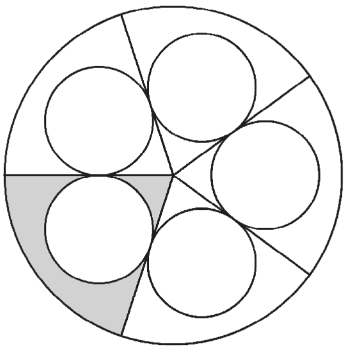


Diagram I shows one large circle and five identical small circles.
Each of the five radii shown is a tangent to two of the small circles.

(i) Describe the symmetry of the diagram. [1]

(ii) The radius of the large circle is R centimetres and the radius of each small circle is r centimetres. Each small circle is equal in area to the shaded region.
Find $R^2 : r^2$. [3]

(b)

Diagram II

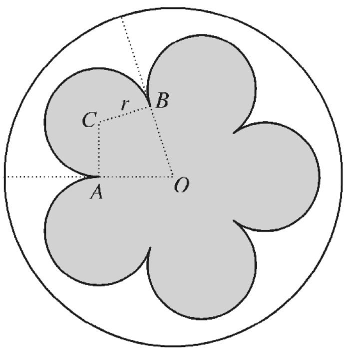


Diagram II shows the same large circle and arcs of the same small circles as in **Diagram I**.
C is the centre of one of the small circles.
This circle touches the adjacent circles at A and B.
O is the centre of the large circle.

(i) Show that reflex $A\hat{C}B = 252°$. [2]

(ii) The perimeter of the shaded region is $k\pi r$ centimetres. Calculate the value of k. [2]

Thinking Process

(a) (i) Understand the definition of line symmetry and rotational symmetry.

(ii) Find the area of the shaded region. Use the given condition to find the required ratio.

(b) (i) To find the reflex $\angle ACB$ ✐ find the interior angles of the quadrilateral $OACB$.

(ii) To find k ✐ find the perimeter of the shaded region. Compare it with the given perimeter.

Solution

(a) (i) The figure has 1 line of symmetry.

(ii) Area of one small circle $= \pi r^2$

The large circle is divided into 5 equal sectors, one of which is shaded.

$\therefore$ area of one sector $= \frac{1}{5}\pi R^2$

area of small circle = area of the shaded region

$$\pi r^2 = \frac{1}{5}\pi R^2 - \pi r^2$$

$$\frac{1}{5}\pi R^2 = 2\pi r^2$$

$$R^2 = 10r^2 \quad \Rightarrow \quad \frac{R^2}{r^2} = \frac{10}{1}$$

$\therefore$ $R^2 : r^2 = 10 : 1$ **Ans.**

(b) (i) $A\hat{O}B = \frac{360°}{5} = 72°$

$O\hat{A}C = O\hat{B}C = 90°$

> OA is tangent ⊥ to radius AC through point of contact A.

In quadriteral $OACB$,

$O\hat{A}C + A\hat{C}B + O\hat{B}C + A\hat{O}B = 360°$

$90° + A\hat{C}B + 90° + 72° = 360°$

$A\hat{C}B = 360° - 90° - 90° - 72°$

$= 108°$

$\therefore$ reflex $A\hat{C}B = 360° - 108°$

$= 252°$ **Shown.**

(ii) Length of arc $A\hat{C}B = \frac{252°}{360°} \times 2\pi r = \frac{7}{5}\pi r$

Total length of 5 arcs $= 5(\frac{7}{5}\pi r) = 7\pi r$

perimeter of the shaded region $= k\pi r$

$\Rightarrow$ $7\pi r = k\pi r$

$\Rightarrow$ $k = 7$ **Ans.**

Topic 17

Vectors in Two Dimensions

1 *(J2011/P1/Q5)*

$\mathbf{c} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{d} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

(a) Calculate $2\mathbf{c} - \mathbf{d}$. [1]

(b) Calculate $|\mathbf{d}|$. [1]

Thinking Process

(a) Perform the required calculation.

(b) Use $|\mathbf{a}| = \sqrt{x^2 + y^2}$, where $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$

Solution

(a) $2\mathbf{c} - \mathbf{d} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 10 \end{pmatrix}$ **Ans.**

(b) $|\mathbf{d}| = \sqrt{(8)^2 + (-6)^2}$

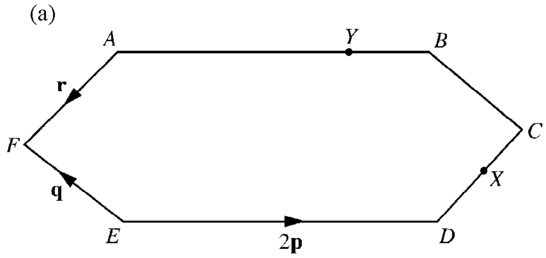
$= \sqrt{64 + 36}$

$= \sqrt{100}$

$= 10$ **Ans.**

2 *(J2011/P2/Q7)*

(a)



In the diagram, *ABCDEF* is a hexagon with rotational symmetry of order 2.

$\vec{ED} = 2\mathbf{p}$, $\vec{EF} = \mathbf{q}$ and $\vec{AF} = \mathbf{r}$.

X is the midpoint of *CD* and *Y* is the point on *AB* such that *AY* : *YB* is 3 : 1.

(i) How many lines of symmetry does *ABCDEF* have? [1]

(ii) Express, as simply as possible, in terms of one or more of the vectors **p**, **q** and **r**,

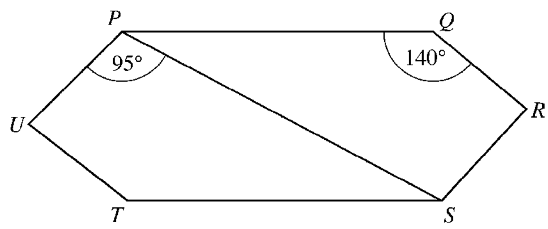
(a) $\vec{EA}$, [1]

(b) $\vec{FC}$, [1]

(c) $\vec{FY}$, [1]

(d) $\vec{YX}$. [1]

(b)



PQRSTU is a similar hexagon to *ABCDEF*.

$U\hat{P}S = 95°$ and $P\hat{Q}R = 140°$.

Find

(i) $Q\hat{P}S$, [1]

(ii) $P\hat{S}R$, [1]

(iii) $P\hat{U}T$. [1]

Thinking Process

(a) (i) Understand the definition of line symmetry.

(ii) (a) $\vec{EA} = \vec{EF} + \vec{FA}$

(b) $\vec{FC} = \vec{FE} + \vec{ED} + \vec{DC}$

(c) $\vec{FY} = \vec{FA} + \vec{AY}$. Note that $\frac{AY}{YB} = \frac{3}{1}$

(d) Find $\vec{YB}$, $\vec{CX}$ and then perform vector addition.

(b) (i) Note that $\angle UPQ = \angle PQR$.

(ii) $\angle PSR = \angle SPU$ (alternats $\angle$s).

(iii) Apply, sum of angles in a quadrilateral.

Solution

(a) (i) Lines of symmetry = 2 **Ans.**

(ii) (a) $\vec{EA} = \vec{EF} + \vec{FA}$

$= \mathbf{q} - \mathbf{r}$ **Ans.**

(b) $\vec{CD} = \vec{AF} = \mathbf{r}$

$\vec{FC} = \vec{FE} + \vec{ED} + \vec{DC}$

$= -\mathbf{q} + 2\mathbf{p} - \mathbf{r}$ **Ans.**

(c) $\frac{\vec{AY}}{\vec{YB}} = \frac{3}{1}$

$\Rightarrow \frac{\vec{AY}}{\vec{AB}} = \frac{3}{4} \Rightarrow \vec{AY} = \frac{3}{4}\vec{AB}$

$\vec{FY} = \vec{FA} + \vec{AY}$

$= -\mathbf{r} + \frac{3}{4}\vec{AB}$

$= -\mathbf{r} + \frac{3}{4}(2\mathbf{p})$

$= -\mathbf{r} + \frac{3}{2}\mathbf{p}$ **Ans.**

(d) $\vec{YB} = \frac{1}{4}\vec{AB}$, $\vec{BC} = \vec{FE}$, $\vec{CX} = \frac{1}{2}\vec{CD}$

$\vec{YX} = \vec{YB} + \vec{BC} + \vec{CX}$

$= \frac{1}{4}\vec{AB} + \vec{FE} + \frac{1}{2}\vec{CD}$

$= \frac{1}{4}(2\mathbf{p}) + (-\mathbf{q}) + \frac{1}{2}(\mathbf{r})$

$= \frac{1}{2}\mathbf{p} - \mathbf{q} + \frac{1}{2}\mathbf{r}$ **Ans.**

(b) (i) $U\hat{P}Q = P\hat{Q}R = 140°$ (interior ∠s)

$Q\hat{P}S = 140° - 95° = 45°$ **Ans.**

(ii) $P\hat{S}R = S\hat{P}U = 95°$ (alternate ∠s) **Ans.**

(iii) $T\hat{S}P = Q\hat{P}S = 45°$ (alternate ∠s)

In quadrilateral $PSTU$,

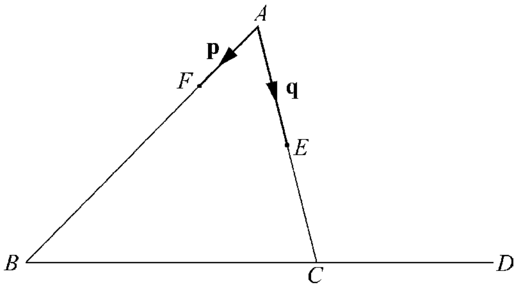
$U\hat{P}S + P\hat{U}T + U\hat{T}S + T\hat{S}P = 360°$

$95° + P\hat{U}T + 140° + 45° = 360°$

$P\hat{U}T = 360° - 280°$

$= 80°$ **Ans.**

3 *(N2011/P1/Q28)*



In the diagram, F is the point on AB where $AF = \frac{1}{4}AB$. E is the midpoint of AC.

$\vec{AF} = \mathbf{p}$ and $\vec{AE} = \mathbf{q}$.

(a) Express, in terms of **p** and **q**,

(i) $\vec{FE}$, [1]

(ii) $\vec{BC}$. [1]

(b) D is the point on BC produced such that $BD = kBC$.

(i) Express $\vec{FD}$ in terms of k, **p** and **q**. [1]

(ii) Given that F, E and D are collinear, find the value of k. [2]

Thinking Process

(a) (i) $\vec{FE} = \vec{FA} + \vec{AE}$

(ii) $\vec{BC} = \vec{BA} + \vec{AC}$

(b) (i) $\vec{FD} = \vec{FB} + \vec{BD}$

(ii) Express $\vec{FE}$ as $h\vec{FD}$ since F, E, and D are collinear. Find the value of h and subsequently find k.

Solution

(a) (i) $\vec{FE} = \vec{FA} + \vec{AE}$

$= \mathbf{q} - \mathbf{p}$ **Ans.**

(ii) $\vec{AF} = \frac{1}{4}\vec{AB} \Rightarrow \vec{AB} = 4\vec{AF} = 4\mathbf{p}$

$\vec{AC} = 2\vec{AE} = 2\mathbf{q}$

$\therefore \vec{BC} = \vec{BA} + \vec{AC}$

$= -4\mathbf{p} + 2\mathbf{q}$ **Ans.**

(b) (i) $\vec{FD} = \vec{FB} + \vec{BD}$

$= 3\mathbf{p} + k\vec{BC}$

$= 3\mathbf{p} + k(-4\mathbf{p} + 2\mathbf{q})$

$= 3\mathbf{p} - 4k\mathbf{p} + 2k\mathbf{q}$

$= \mathbf{p}(3 - 4k) + 2k\mathbf{q}$ **Ans.**

(ii) Since F, E and D are collinear,

$\vec{FE} = h\vec{FD}$, where h is a constant.

$\Rightarrow \quad \mathbf{q} - \mathbf{p} = h(\mathbf{p}(3-4k) + 2k\mathbf{q})$

$-\mathbf{p} + \mathbf{q} = h\mathbf{p}(3-4k) + 2hk\mathbf{q}$

comparing coefficients of $\mathbf{p}$ and $\mathbf{q}$,

$-1 = h(3-4k) \quad \Rightarrow \quad -1 = 3h - 4hk$ ....(1)

$1 = 2hk \quad \Rightarrow \quad k = \frac{1}{2h}$ ....(2)

substitute (2) into (1)

$-1 = 3h - 4h(\frac{1}{2h})$

$-1 = 3h - 2$

$3h = 1$

$h = \frac{1}{3}$

substitute $h = \frac{1}{3}$ into (2)

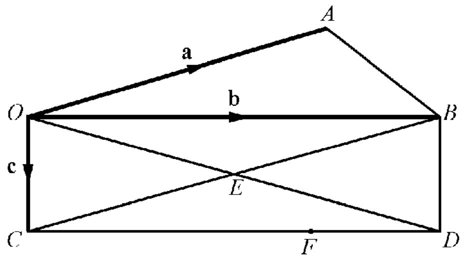
$k = \frac{1}{2(\frac{1}{3})} \quad \Rightarrow \quad k = \frac{3}{2}$ or 1.5 **Ans.**

4 *(J2012/P2/Q7)*

OAB is a triangle and $OBDC$ is a rectangle where OD and BC intersect at E.

F is the point on CD such that $CF = \frac{3}{4}CD$.

$\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$.



(a) Express, as simply as possible, in terms of one or more of the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$,

(i) $\vec{AB}$, [1]

(ii) $\vec{OE}$, [1]

(iii) $\vec{EF}$. [2]

(b) G is the point on AB such that $\vec{OG} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$.

(i) Express $\vec{AG}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer as simply as possible. [1]

(ii) Find $AG : GB$. [1]

(iii) Express $\vec{FG}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.
Give your answer as simply as possible. [2]

Thinking Process

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

(ii) $\vec{OE} = \frac{1}{2}\vec{OD}$ ✐ find $\vec{OD}$

(iii) $\vec{EF} = \vec{EO} + \vec{OC} + \vec{CF}$

(b) (i) $\vec{AG} = \vec{OG} - \vec{OA}$

(ii) To find the ratio ✐ express AG and BG in terms of AB.

(iii) To find $\vec{FG}$ ✐ perform vector addition.

Solution

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \mathbf{b} - \mathbf{a}$ **Ans.**

(ii) $\vec{OE} = \frac{1}{2}\vec{OD}$

$= \frac{1}{2}(\vec{OC} + \vec{CD})$

$= \frac{1}{2}(\mathbf{c} + \mathbf{b})$ **Ans.**

(iii) $\vec{EF} = \vec{EO} + \vec{OC} + \vec{CF}$

$= \vec{EO} + \vec{OC} + \frac{3}{4}\vec{CD}$

$= -\frac{1}{2}(\mathbf{c} + \mathbf{b}) + \mathbf{c} + \frac{3}{4}\mathbf{b}$

$= -\frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{b} + \mathbf{c} + \frac{3}{4}\mathbf{b}$

$= \frac{1}{4}\mathbf{b} + \frac{1}{2}\mathbf{c}$ **Ans.**

(b) (i) $\vec{AG} = \vec{OG} - \vec{OA}$

$= \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \mathbf{a}$

$= \frac{2}{5}\mathbf{b} - \frac{2}{5}\mathbf{a}$

$= \frac{2}{5}(\mathbf{b} - \mathbf{a})$ **Ans.**

(ii) $\vec{AG} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{5}\vec{AB}$

$\Rightarrow \quad \vec{GB} = \frac{3}{5}\vec{AB}$

$\therefore \quad AG : GB = 2 : 3$ **Ans.**

(iii) $\vec{FG} = \vec{FC} + \vec{CO} + \vec{OA} + \vec{AG}$

$= -\frac{3}{4}\vec{CD} + \vec{CO} + \vec{OA} + \vec{AG}$

$= -\frac{3}{4}\mathbf{b} - \mathbf{c} + \mathbf{a} + \frac{2}{5}\mathbf{b} - \frac{2}{5}\mathbf{a}$

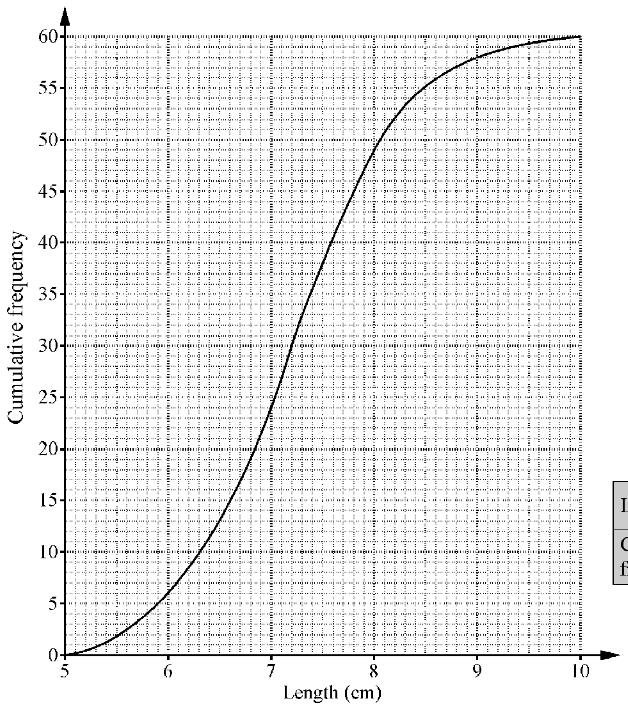
$= \frac{3}{5}\mathbf{a} - \frac{7}{20}\mathbf{b} - \mathbf{c}$ **Ans.**

Topic 18

Statistics

1 (J2012/P2/Q5)

(a) The cumulative frequency graph shows the distribution of the lengths of 60 leaves.



(i) Complete the table to show the distribution of the lengths of the leaves.

Length (l cm)	$5 < l \le 6$	$6 < l \le 7$	$7 < l \le 8$	$8 < l \le 9$	$9 < l \le 10$
Frequency	6	18			2

[1]

(ii) Use the graph to estimate the median. [1]

(iii) Use the graph to estimate the interquartile range. [2]

(iv) One of these leaves is chosen at random. Estimate the probability that it has a length of **more than** 7.5 cm. [2]

(b) The distribution of the widths of these leaves is shown in the table below.

Width (w cm)	$3 < w \le 4$	$4 < w \le 5$	$5 < w \le 6$	$6 < w \le 7$	$7 < w \le 8$	$8 < w \le 9$
Frequency	4	15	20	13	5	3

(i) Calculate an estimate of the mean width. [3]

(ii) Calculate the percentage of leaves with a width of more than 6 cm. [2]

Thinking Process

(a) (i) To complete the table ✐ use the relationship between frequency and cumulative frequency.

(ii) ✐ Find 50% of total frequency and read the corresponding value for the length.

(iii) ✐ Read 75% of frequency and 25% of frequency. Subtract.

(iv) From graph, find the number of leaves that have length more than 7.5 cm. Express them as a fraction of total leaves.

(b) (i) Mean = $\frac{\sum f x}{\sum f}$ ✐ use the mid-class width from each category in the computation.

(ii) Express the number of leaves that have widths greater than 6 cm as a percentage of total frequency.

Solution

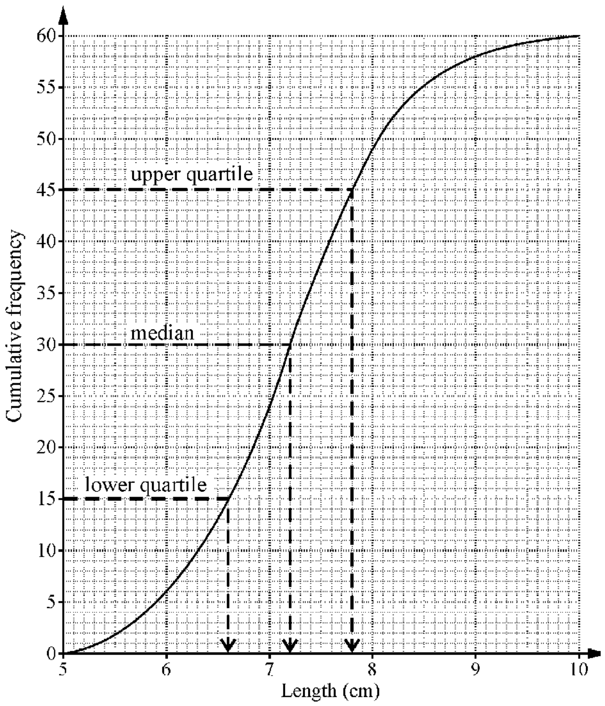
(a) (i) From graph, the cumulative frequencies are:

Length (l cm)	$5 < l \le 6$	$6 < l \le 7$	$7 < l \le 8$	$8 < l \le 9$	$9 < l \le 10$
Cumulative frequency	6	24	49	58	60

$\therefore$ for class $7 < l \le 8$, frequency $= 49 - 24$
$= 25$ **Ans.**

for class $8 < l \le 9$, frequency $= 58 - 49$
$= 9$ **Ans.**

(ii) Median length = 7.2 cm **Ans.**



(iii) Interquartile range
= upper quartile − lower quartile
= 7.8 − 6.6 = 1.2 cm **Ans.**

(iv) Number of leaves with length of more than 7.5 cm = 60 − 38 = 22

$P(\text{length } > 7.5 \text{ cm}) = \frac{22}{60} = \frac{11}{30}$ **Ans.**

(b) (i)

Width (w cm)	Midpoint (x)	Frequency (f)	fx
$3 < w \le 4$	3.5	4	14
$4 < w \le 5$	4.5	15	67.5
$5 < w \le 6$	5.5	20	110
$6 < w \le 7$	6.5	13	84.5
$7 < w \le 8$	7.5	5	37.5
$8 < w \le 9$	8.5	3	25.5
		$\sum f = 60$	$\sum fx = 339$

Mean width $= \frac{\sum fx}{\sum f}$

$= \frac{339}{60} = 5.65$ cm **Ans.**

(ii) Number of leaves with a width of more than 6 cm = 13 + 5 + 3 = 21

$\frac{21}{60} \times 100 = 35\%$ **Ans.**

2 *(N2012/P1/Q9)*

The number of goals scored by some football teams during one weekend was recorded.
The table shows the results.

Number of goals scored	0	1	2	3	4
Number of teams	x	1	5	4	2

(a) If the mode is 0, find the smallest possible value of x. [1]

(b) If the median is 1, find the value of x. [1]

Thinking Process

(a) Since mode is 0, the value of x must be greater then the highest frequency of x.

(b) Remember that the median is the data in the middle position.

Solution

(a) Smallest possible value of $x = 6$ **Ans.**

(b) $x = 11$ **Ans.**

3 *(N2012/P2/Q6)*

The heights of 150 children are measured.
The results are summarised in the table.

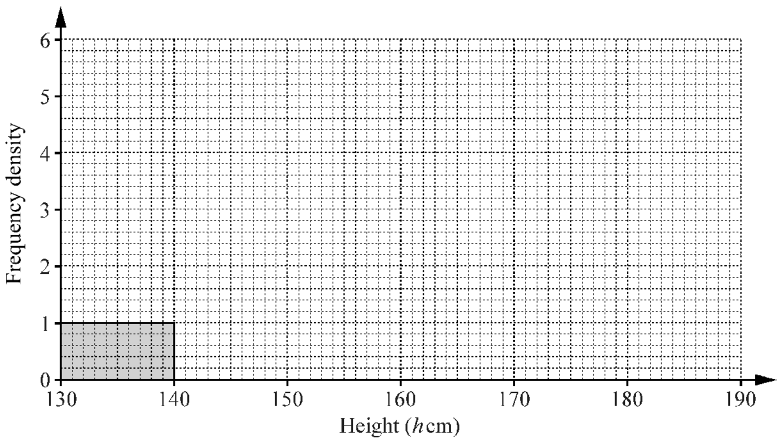
Height (h cm)	$130 < h \le 140$	$140 < h \le 150$	$150 < h \le 155$	$155 < h \le 160$	$160 < h \le 170$	$170 < h \le 190$
Frequency	10	30	20	30	35	25

(a) Calculate an estimate of the mean height. [3]

(b) (i) One child is chosen at random.
Find the probability that this child has a height greater than 160 cm. [1]

(ii) Two children are chosen at random without replacement.
Find the probability that the height of one child is greater than 160 cm and the height of the other is 150 cm or less. [2]

(c) Complete the histogram to represent the information in the table.



[3]

Thinking Process

(a) Mean $= \frac{\sum fx}{\sum f}$

(b) (i) Express the number of children with height greater than 160 cm as a fraction of total number of children.

(ii) 2 × P(more than 160 cm) × P(150 cm or less).

(c) Note that the distribution is of unequal widths. Find the width of each interval and hence frequency density of each interval.

Solution

(a)

height (h cm)	Midpoint (x)	Frequency (f)	fx
$130 \le h < 140$	135	10	1350
$140 \le h < 150$	145	30	4350
$150 \le h < 155$	152.5	20	3050
$155 \le h < 160$	157.5	30	4725
$160 \le h < 170$	165	35	5775
$170 \le h < 190$	180	25	4500
		$\sum f = 150$	$\sum fx = 23750$

Mean height $= \frac{\sum fx}{\sum f}$

$= \frac{23750}{150} = 158.33$

≈ 158 cm **Ans.**

(b) (i) No. of children with height greater than 160 cm $= 35 + 25 = 60$

P(ht. of child $>$160 cm)

$= \frac{60}{150} = \frac{2}{5}$ **Ans.**

(ii) No. of children with height 150 cm or less

$= 10 + 30 = 40$

P(ht. of one child is $>$ 160 cm and ht. of other is 150 cm or less) $= \left(\frac{60}{150} \times \frac{40}{149}\right) + \left(\frac{40}{150} \times \frac{60}{149}\right)$

$= \frac{32}{149}$ **Ans.**

(c)

Height (h cm)	Width	Frequency	Frequency density
$130 \le h < 140$	10	10	$\frac{10}{10} = 1$
$140 \le h < 150$	10	30	$\frac{30}{10} = 3$
$150 \le h < 155$	5	20	$\frac{20}{5} = 4$
$155 \le h < 160$	5	30	$\frac{30}{5} = 6$
$160 \le h < 170$	10	35	$\frac{35}{10} = 3.5$
$170 \le h < 190$	20	25	$\frac{25}{20} = 1.25$