To provide an idea about what this book contains, only few pages taken randomly from the book are shown here.

KEY•POINTS SERIES

ABOUT MATHEMATICS KEY•POINTS

This book covers the newly revised syllabus for GCE 'O' LEVEL MATHEMATICS. Aiming to provide students with an effective yet easy-to-follow guide, each chapter in this book consists of the following learning skills:

About (NExT)
Nice-aim Target Guide

i. To unlock the contents in the **OVERVIEW OF THE TOPIC**.

or

ii. To search for next subject.

Overview Of The Topic

This table shows the structure and all the key areas of a topic.

Objectives

Objectives based on the latest syllabus are given in this part. Students should make sure that they are able to meet all of the expectations before taking the examination.

Key Points

This part is featured by concise study notes. All key concepts and formulae are presented to help students consolidate their knowledge learnt in class.

Stop & Think

These are useful learning-type questions provided after each subtopic to help students identify and enhance the learning of key concepts.

Challenging Questions with Worked Solutions

At the end of each chapter, challenging exam-type questions enable students to have constant practice in order for them to apply the concepts that they have learnt. Fully worked out solutions are also provided to show the correct way of presenting workings and answers.

The Publisher

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TOPIC 12

MENSURATION

OVERVIEW OF THE TOPIC

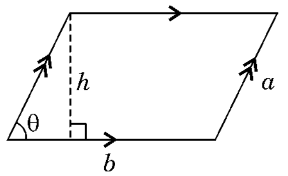
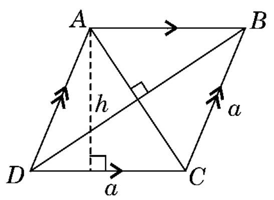
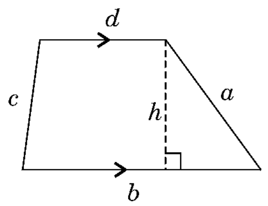
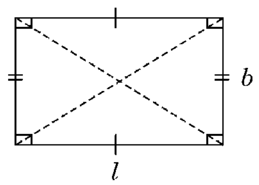
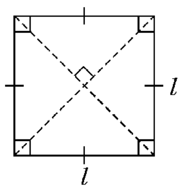
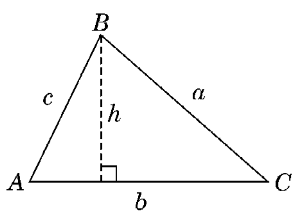
MENSURATION *2 key areas*	PERIMETER AND AREA	• Triangles and quadrilaterals
		• Circles
	SURFACE AREA AND VOLUME OF SOLIDS	• Cube and cuboid
		• Prism
		• Cylinder
		• Cone
		• Pyramid
		• Sphere

PERIMETER AND AREA

Objectives: Find the perimeter and area of triangles, quadrilaterals and circles. Solve problems involving the perimeter and area of triangles, quadrilaterals and circles.

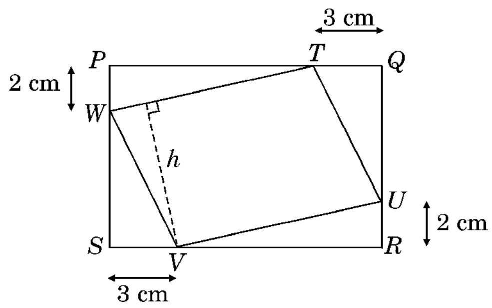
Triangles and quadrilaterals

- ☑ The ***perimeter*** of any closed figure is the total length of all the sides.
- ☑ To find ***area*** of quadrilaterals and triangles, apply formulae:

	**Parallelogram** ① Perimeter $= a+b+a+b = 2(a+b)$ ② Area $=$ base $\times$ height $= b\times h$ (or Area $= ab\sin\theta$)
	**Rhombus** ① Perimeter $= a+a+a+a = 4a$ ② Area $=$ base $\times$ height $= a\times h$ $\left(\text{or Area} = \frac{1}{2}\times AC\times BD\right)$
	**Trapezium** ① Perimeter $= a+b+c+d$ ② Area $= \frac{1}{2}\times$(sum of // sides)$\times$ height $= \frac{1}{2}\times(b+d)\times h$
	**Rectangle** ① Perimeter $= 2(l+b)$ ② Area $=$ length $\times$ breadth $= l\times b$
	**Square** ① Perimeter $= 4l$ ② Area $= l\times l = l^2$
	**Triangle** ① Perimeter $= a+b+c$ ② Area $= \frac{1}{2}\times$ base $\times$ height $= \frac{1}{2}\times b\times h$ $\left(\text{or Area} = \frac{1}{2}ab\sin C\right)$

STOP AND THINK

[Q] The rectangle $PQRS$ measures 12 cm by 8 cm. Points T, U, V and W are on the sides with measurements, in centimetres, as shown. Find the area, in square centimetres, of $TUVW$. Find also the height, h, of the parallelogram $TUVW$.



SOLUTION

$PT = VR = 12 - 3 = 9$ cm
$QU = SW = 8 - 2 = 6$ cm

$$\begin{aligned}\text{Area of } TUVW &= \text{Area of } PQRS - (2 \times \text{Area of } \Delta PWT) - (2 \times \text{Area of } \Delta TQU)\\ &= (12 \times 8) - \left(2 \times \frac{1}{2} \times 2 \times 9\right) - \left(2 \times \frac{1}{2} \times 3 \times 6\right)\\ &= 96 - 18 - 18\\ &= 60 \text{ cm}^2 \quad \textbf{(Ans)}\end{aligned}$$

Apply Pythagoras' theorem, $WT = \sqrt{9^2 + 2^2} = \sqrt{85}$ cm

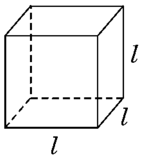
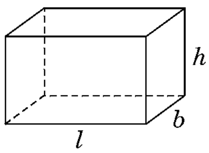
$$\begin{aligned}\text{Area of } TUVW &= WT \times h\\ \Rightarrow \quad 60 &= \sqrt{85} \times h\\ \therefore \quad h &= \frac{60}{\sqrt{85}}\\ &\approx 6.51 \text{ cm} \ (3 \text{ sf}) \quad \textbf{(Ans)}\end{aligned}$$

SURFACE AREA AND VOLUME OF SOLIDS

Objective: Find the surface area and volume of cube, cuboid, prism, cylinder, cone, pyramid, sphere and hemisphere.

Cube and cuboid

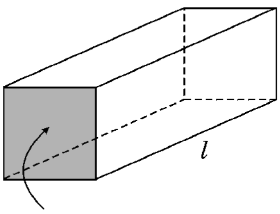
- ☑ All solids are 3-dimensional.
- ☑ A ***cube*** is a solid with equal length, breadth and height, so it has 6 common square surfaces.
- ☑ If the 3 sides are not the same, the solid is a ***cuboid***.
- ☑ To find ***total surface area*** and ***volume*** of cube and cuboid:

	**Cube** ① Total surface area $= 6 \times$ area of square $= 6l^2$ ② Volume $=$ length $\times$ length $\times$ length $= l^3$
	**Cuboid** ① Total surface area $= 2lb + 2lh + 2bh = 2(lb + lh + bh)$ ② Volume $=$ length $\times$ breath $\times$ height $= lbh$

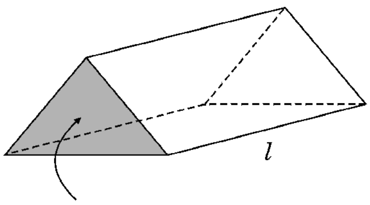
Prism

- ☑ Cube and cuboid are two types of prisms.
- ☑ A ***prism*** is a solid that has the same cross-section running through it.

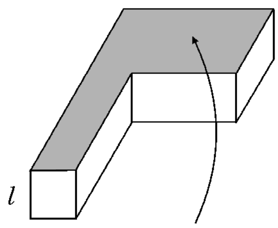
EXAMPLE



cross-sectional area, A



cross-sectional area, A



cross-sectional area, A

- ☑ To find ***volume*** and ***total surface area***:

Prism

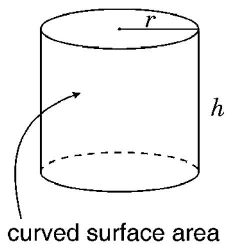
① Volume $=$ area of cross-section $\times$ length $= Al$

② Total surface area $= (2 \times \text{area of cross-section}) + (\text{perimeter of cross-section} \times \text{length})$

Cylinder

☑ A ***circular cylinder*** has a top and a bottom which are circles of equal size. Hence, it is a type of prisms with a circular cross-section.

☑ To find ***volume***, ***curved surface area*** and ***total surface area***:



Circular Cylinder

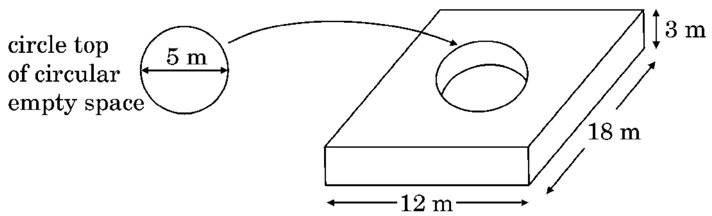
① Volume = area of circle × height $= \pi r^2 h$

② Curved surface area = perimeter of circle × height $= 2\pi rh$

③ Total surface area = curved surface area + (2 × area of circle) $= 2\pi rh + 2\pi r^2$

STOP AND THINK

[Q] Find the volume and total surface area of the solid shown, taking $\pi = 3.14$.



SOLUTION

Volume of solid = volume of cuboid − volume of cylinder

$= (12 \times 18 \times 3) - (3.14 \times 2.5 \times 2.5 \times 3)$

$= 648 - 58.875$

$= 589.125$

$= 589 \text{ m}^3$ (3 sf) **(Ans)**

Total surface area ← [front & back + left & right + top & bottom without circles + internal side wall of circular empty space]

$= 2(12 \times 3) + 2(18 \times 3) + 2[(12 \times 18) - (3.14 \times 2.5 \times 2.5)] + (2 \times 3.14 \times 2.5 \times 3)$

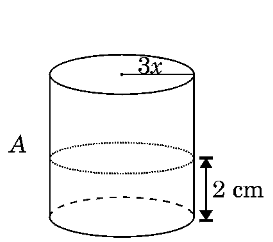
$= 72 + 108 + 392.75 + 47.1$

$= 619.85$

$= 620 \text{ m}^2$ (3 sf) **(Ans)**

STOP AND THINK

[Q] Two cylindrical jars A and B have radii of $3x$ centimetres and x centimetres respectively. Initially, cylinder B is empty and cylinder A contains water to a depth of 2 cm. If this water is all poured into cylinder B, calculate the height it will reach.



SOLUTION

Let height of water in cylinder B be h.

Volume of water in cylinder A $= \pi \times (3x)^2 \times 2$
$= 18\pi x^2 \text{ cm}^3$

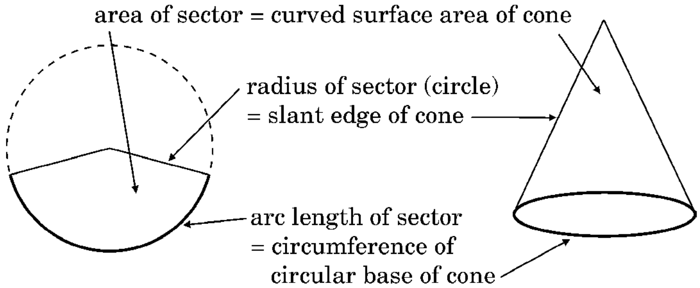
Volume of water from A in cylinder $B = \pi x^2 h$

$\Rightarrow \quad 18\pi x^2 = \pi x^2 h$

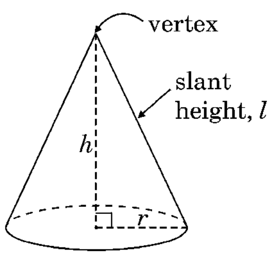
$\therefore \quad h = \dfrac{18\pi x^2}{\pi x^2}$
$= 18 \text{ cm}$ **(Ans)**

Cone

- ☑ A ***circular cone*** is a solid with a circular base and a vertex.
- ☑ The curved surface is formed from sector of a circle.

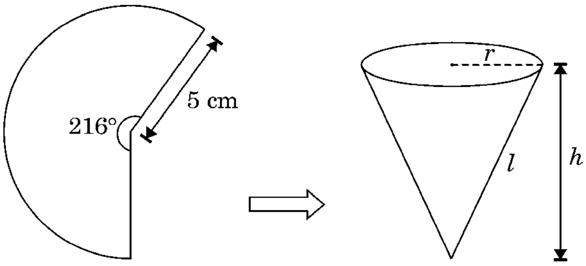


- ☑ To find ***volume***, ***curved surface area*** and ***total surface area***:

	**Circular Cone** ① Volume $= \frac{1}{3}\pi r^2 h$ ② Curved surface area $= \pi r l$ ③ Total surface area $=$ curved surface area + area of circle $= \pi r l + \pi r^2$

STOP AND THINK

[Q] A piece of paper in the form of a circular sector is made to form a right circular cone. The radius of the sector is 5 cm. The angle at the centre is 216°.



(i) Write down the length of the slant edge l of the cone formed.

(ii) Find the radius r of the circular base of the cone.

(iii) The curved surface area of the cone can be obtained using the formula πrl where r is the base radius and l the slant height of the cone. Find the curved surface area leaving your answer in terms of π.

Suggest another way of finding the curved surface area and show that the same answer is obtained using the suggested method.

(iv) Taking π to be 3.142, find the capacity of the cone.

SOLUTION

(i) Slant edge of cone = radius of sector

$\therefore\ l = 5\text{ cm}$ **(Ans)**

(ii) Arc length of sector $= \frac{216°}{360°} \times 2 \times \pi \times 5$

$= 6\pi\text{ cm}$

Circumference of circular base of cone = arc length of sector

$\Rightarrow \quad 2\pi r = 6\pi$

$\therefore\ r = 3\text{ cm}$ **(Ans)**

(iii) Curved surfaced area of cone $= \pi \times 3 \times 5$

$= 15\pi\text{ cm}^2$ **(Ans)**

Another way of finding curved surface area is to find the area of the sector.

Area of sector $= \frac{216°}{360°} \times \pi \times 5 \times 5$

$= 15\pi\text{ cm}^2$ (shown)

(iv) Apply Pythagoras' theorem, height of cone, $h = \sqrt{5^2 - 3^2}$

$= 4\text{ cm}$

Capacity of cone = volume of cone

$= \frac{1}{3} \times \pi \times 3^2 \times 4$

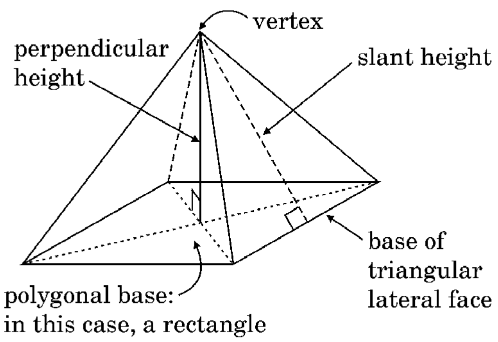
$= 3.142 \times 12$

$= 37.704$

$= 37.7\text{ cm}^3$ (3 sf) **(Ans)**

Pyramid

- ☑ A ***regular pyramid*** is a solid that has a regular polygonal base with a perpendicular vertex and slant triangular lateral faces.
- ☑ The polygonal base can be a triangle, a square, a rectangle, etc.
- ☑ To find ***volume*** and ***total surface area***:



Regular Pyramid

① Volume $= \frac{1}{3} \times$ base area $\times$ height

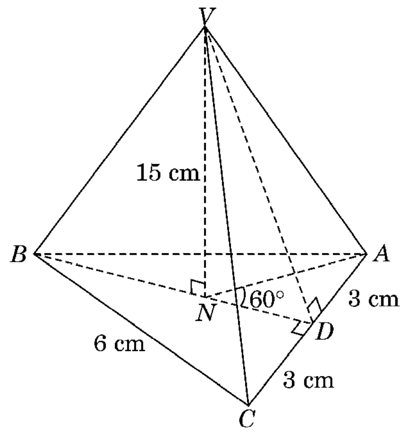
② Total surface area
= base area
+ total area of slant Δ faces

area of slant Δ face $= \frac{1}{2} \times$ base $\times$ slant height

STOP AND THINK

[Q] The base ABC of the pyramid is an equilateral triangle whose sides are 6 cm long.

Given that $\angle AND = 60°$ and the height, VN, of the pyramid is 15 cm, find the volume of the pyramid and total area of the slant triangular faces of the pyramid.



SOLUTION

$$\text{Volume of pyramid} = \frac{1}{3} \times \left(\frac{1}{2} \times 6 \times 6 \times \sin 60°\right) \times 15$$
$$= 77.94$$
$$\approx 77.9 \text{ cm}^3 \quad (3 \text{ sf}) \quad \textbf{(Ans)}$$

$$ND = \frac{3}{\tan 60°} = \frac{3}{\sqrt{3}} = \sqrt{3} \text{ cm}$$

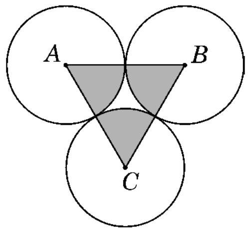
$$VD = \sqrt{15^2 + ND^2} = \sqrt{225 + 3} = \sqrt{228} \text{ cm}$$

$$\text{Total area of slant triangular faces} = 3\left(\frac{1}{2} \times 6 \times \sqrt{228}\right)$$
$$= 135.89$$
$$\approx 136 \text{ cm}^2 \quad (3 \text{ sf}) \quad \textbf{(Ans)}$$

CHALLENGING QUESTIONS

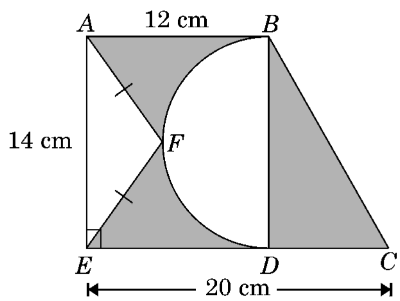
1. 1.89 litres of pure orange juice was packed in a cuboid container of base 7 cm by 9 cm and height 32 cm. ($1\ell = 1000\ \text{cm}^3$)
   (a) Find the volume of the cuboid container.
   (b) What percentage of the volume of the cuboid is filled with the juice?

2.



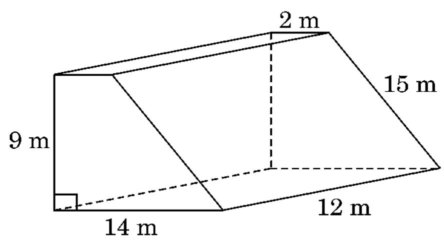
A, B and C are the centres of 3 congruent circles. Calculate the area of the shaded part of the circles if the radius of each circle is 10 cm. (Take $\pi = 3.14$)

3.



The figure shows a semicircle BFD and an isosceles triangle AFE enclosed in a trapezium $ABCDE$ of parallel sides $AB = 12$ cm and $EDC = 20$ cm. Given that the height of the trapezium is 14 cm, $AF = FE$ and $\angle AED = 90°$, calculate the area of the shaded regions. (Take $\pi = \frac{22}{7}$)

4. Calculate the volume and the surface area of the following prism.



WORKED SOLUTIONS

1. (a) Volume of cuboid container
$= 7 \times 9 \times 32$
$= 2016 \text{ cm}^3$ **(Ans)**

(b) $1.89\ell = 1.89 \times 1000$
$= 1890 \text{ cm}^3$

Percentage of volume filled with orange juice
$= \frac{1890}{2016} \times 100\%$
$= 93.75\%$ **(Ans)**

2. Since $\triangle ABC$ is an equilateral triangle, each of the sector has an angle of 60°.

Area of the shaded region $= 3 \times \left(\frac{60°}{360°} \times 3.14 \times 10^2\right)$
$= 157 \text{ cm}^2$ **(Ans)**

3. Area of trapezium $= \frac{1}{2} \times (12+20) \times 14$
$= 224 \text{ cm}^2$

Radius of semicircle $= \frac{14}{2} = 7 \text{ cm}$

Area of semicircle $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$
$= 77 \text{ cm}^2$

Height of $\triangle AEF = 12 - 7 = 5 \text{ cm}$

Area of $\triangle AEF = \frac{1}{2} \times 14 \times 5$
$= 35 \text{ cm}^2$

Area of the shaded region $= 224 - 77 - 35$
$= 112 \text{ cm}^2$ **(Ans)**

4. Volume of prism = area of cross-section × length
$= \left(\frac{1}{2} \times (2+14) \times 9\right) \times 12$
$= 72 \times 12$
$= 864 \text{ m}^3$ **(Ans)**

Surface area of prism $= 2(72) + 12(9+14+15+2)$
$= 144 + 480$
$= 624 \text{ m}^2$ **(Ans)**

5. 1 cm = 10 mm

(a) Volume of water the mug can hold
$= (3.14 \times 4 \times 4 \times 8) \text{ cm}^3$
$= 401.92 \text{ cm}^3$ **(Ans)**

(b) Depth of water $= \frac{\text{volume of water}}{\pi r^2}$
$= \left(\frac{320}{3.14 \times 4 \times 4}\right) \text{cm}$
$\approx 6.37 \text{ cm}$
$\approx 64 \text{ mm}$ (nearest mm) **(Ans)**

(c) Total surface area

$=$

Curved surface area: outer & inner	+	Base area: outer & inner	+	Top area: outer less inner

$$= \left[\begin{array}{l} (2 \times 3.14 \times 45 \times 85) + (2 \times 3.14 \times 40 \times 80) \\ + (3.14 \times 45^2) + \cancel{(3.14 \times 40^2)} \\ + (3.14 \times 45^2) - \cancel{(3.14 \times 40^2)} \end{array}\right] \text{mm}^2$$

$= (24021 + 20096 + 12717) \text{ mm}^2$
$= 56834 \text{ mm}^2$ **(Ans)**

6. (a) Volume of pyramid $TABCD$
$= \frac{1}{3} \times 20 \times 20 \times 60$
$= 8000 \text{ cm}^3$ **(Ans)**

(b) $TM = \frac{1}{2}TE$
$= \frac{1}{2}\left(\frac{1}{2}TA\right)$
$= \frac{1}{4}TA$
$\Rightarrow TM : TA = 1 : 4$

$\frac{\text{Volume of pyramid } TMPQR}{\text{Volume of pyramid } TABCD} = \left(\frac{1}{4}\right)^3$
$\Rightarrow \frac{\text{Volume of pyramid } TMPQR}{8000} = \frac{1}{64}$
$\therefore$ Volume of pyramid $TMPQR = \frac{8000}{64}$
$= 125 \text{ cm}^3$ **(Ans)**

(c) $TM : TE = 1 : 2$

$\frac{\text{Volume of pyramid } TMPQR}{\text{Volume of pyramid } TEFGH} = \left(\frac{1}{2}\right)^3$
$\Rightarrow \frac{125}{\text{Volume of pyramid } TEFGH} = \frac{1}{8}$
$\therefore$ Volume of pyramid $TEFGH = 8 \times 125$
$= 1000 \text{ cm}^3$

Volume of middle portion $= 1000 - 125$
$= 875 \text{ cm}^3$ **(Ans)**