To provide an idea about what this book contains, only few pages taken randomly from the book are shown here.

GCE 'O' Level Mathematics (Topical)

CONTENTS

Syllabus

Topic 4

Algebraic Expressions and Manipulations

1 *(J96/P1/Q14)*

Questions are not shown in Preview

Question 1

Thinking Process

(a) use $a^2 - b^2 = (a-b)(a+b)$.

(b) To factorise the expression ✐ find the common factor of the first two terms and the common factor of the last two terms.

Solution

(a) $16 - 9x^2 = 4^2 - (3x)^2$

$= (4-3x)(4+3x)$ **Ans.**

(b) $6ab - 2ad - 3bc + cd = 2a(3b-d) - c(3b-d)$

$= (2a-c)(3b-d)$ **Ans.**

2 *(D96/P1/Q3)*

Questions are not shown in Preview

Question 2

Thinking Process

To find the sum in terms of x ✐ find an expression for each of the two even numbers in terms of x.

Solution

.... x, $x+1$, $x+2$, $x+3$, ...

↑ odd, ↑ even, ↑ odd, ↑ even

sum of next two even numbers $= (x+1)+(x+3)$

$= 2x+4$ **Ans.**

3 *(D96/P2/Q2)*

Questions are not shown in Preview

Question 3

Thinking Process

(c) (i) To calculate the total cost in (i) ✐ calculate the variable charge.

(ii) To find a formula for C in terms of n ✐ follow the steps in (i).

(iii) To find the greatest number of words Arthur can use ✐ solve the inequality $C < 300$.

Solution *with* TEACHER's COMMENTs

(a) $2p - 5 = 4 - 3(p+2)$

$2p - 5 = 4 - 3p - 6$

$5p = 4 - 6 + 5$

$= 3$

$p = \frac{3}{5}$ **Ans.**

(b) $y = \frac{A+2x}{x}$

$= \frac{A}{x} + 2$

$\frac{A}{x} = y - 2$

$\frac{x}{A} = \frac{1}{y-2}$

$x = \frac{A}{y-2}$ **Ans.**

Alternatively, you can do it this way:

$y = \frac{A+2x}{x}$

$xy = A + 2x$

$xy - 2x = A$

$x(y-2) = A$

$x = \frac{A}{y-2}$

(c) (i) Fixed charge = 50 cents

Variable charge = 15 × 11 cents

= 165 cents

Total cost = 50 + 165 cents

= 215 cents **Ans.**

(ii) Fixed charge = 50 cents

Variable charge = 15 × n cents

= $15n$ cents

Total cost, C = 50 + $15n$ cents **Ans.**

(iii) $C < 300$

$50 + 15n < 300$

$15n < 250$

$n < 16.7$

$\therefore$ the greatest number of words = 16 **Ans.**

4 *(J97/P1/Q18)*

Questions are not shown in Preview

Question 4

Thinking Process

(a) ✐ Apply the formula $a^2 - b^2 = (a - b)(a + b)$ ✐ take out the common factor.

(b) ✐ Use inspection on coefficients.

Solution *with* TEACHER's COMMENTs

(a) $5 - 45t^2$

$= 5(1 - 9t^2)$

$= 5\left[1^2 - (3t)^2\right]$

$= 5(1 - 3t)(1 + 3t)$

> Important to note that $1 = 1^2$ and $9t^2 = (3t)^2$ and then apply $a^2 - b^2 = (a - b)(a + b)$ to factorize the expression further.

(b) Note that:

	3		−2
	2		1
	↓	↓	↓
	6	(−4 + 3)	−2
ie.	6	−1	−2

$\therefore 6p^2 - p - 2$

$= (3p - 2)(2p + 1)$

5 *(J97/P1/Q21)*

Questions are not shown in Preview

Question 5

Thinking Process

(a) To express t in terms of s ✐ express $t - 2$ in terms of s.

(b) To express $\dfrac{4}{2x-1} - \dfrac{3}{5x+6}$ as a single fraction ✐ write both fractions with $(2x - 1)(5x + 6)$ as denominator and simplify the numerators of the fractions.

Solution

(a) $s = \dfrac{3}{t-2}$

$\Rightarrow s(t - 2) = 3$

$\Rightarrow t - 2 = \dfrac{3}{s}$

$\Rightarrow t = \dfrac{3}{s} + 2 = \dfrac{3 + 2s}{s}$ **Ans.**

(b) $\dfrac{4}{2x-1} - \dfrac{3}{5x+6}$

$= \dfrac{4(5x+6)}{(2x-1)(5x+6)} - \dfrac{3(2x-1)}{(2x-1)(5x+6)}$

$= \dfrac{(20x+24) - (6x-3)}{(2x-1)(5x+6)}$

$= \dfrac{14x+27}{(2x-1)(5x+6)}$ **Ans.**

6 *(D97/P1/Q12)*

Questions are not shown in Preview

Question 6

Thinking Process

(a) To factorize $2\pi r^2 + 2\pi rh$ ✐ consider the common factor of $2\pi r^2$ and $2\pi rh$.

(b) To factorize $ac - 3c + 2ab - 6b$ ✐ By inspection or consider the common factor in $ac - 3c$ and $2ab - 6b$ respectively.

Solution

(a) $2\pi r$ is the common factor of $2\pi r^2$ and $2\pi rh$,

$\therefore 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$ **Ans.**

(b) To factorize $ac - 3c + 2ab - 6b$, consider the expressions $ac - 3c$ and $2ab - 6b$ separately and attempt to find if common factor can be found in them.

$ac - 3c$ has a common factor $c \Rightarrow ac - 3c = c(a - 3)$

$2ab - 6b$ has a common factor $2b \Rightarrow 2ab - 6b = 2b(a - 3)$

Now, $ac - 3c + 2ab - 6b = c(a - 3) + 2b(a - 3)$ which has a common factor $(a - 3)$.

$\therefore ac - 3c + 2ab - 6b = (c + 2b)(a - 3)$

$= (a - 3)(2b + c)$ **Ans.**

Topic 18

Vectors in Two Dimensions

21 *(D2004/P1/Q13)*

Questions are not shown in Preview

Question 21

Thinking Process

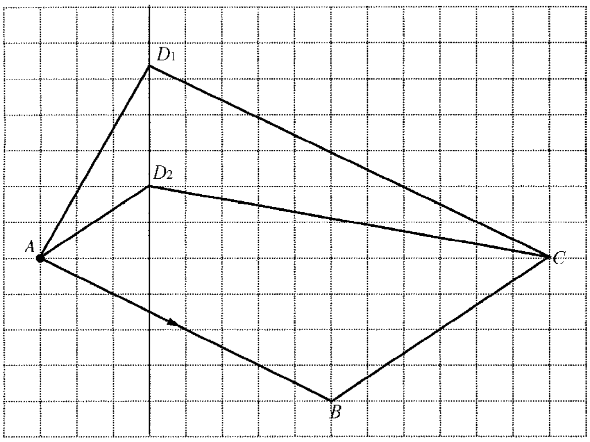
(a) Apply vector addition: $\vec{AC} = \vec{AB} + \vec{BC}$.

(b) For *ABCD* to be a trapezium, *CD* must be // to *AB* or *AD* must be // to *BC*.

Solution

(a) $\vec{AC} = \vec{AB} + \vec{BC} = \begin{pmatrix} 8 \\ -4 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \end{pmatrix}$

(b) $h = 5\frac{1}{2}$ or 2.



22 *(D2005/P2/Q11)*

Questions are not shown in Preview

Question 22

Thinking Process

(a) (i) $\vec{DO} = \vec{OA}$.

(ii) $\vec{AB} = \vec{AO} + \vec{OB}$.

(iii) $\vec{DB} = \vec{DO} + \vec{OB}$.

(b) Since *ABCDEF* is a regular hexagon, *OAB* is an equilateral Δ . $\therefore OB = OA = AB$.

(c) (i) (a) $\vec{AX} = \vec{OX} - \vec{OA}$.

(b) $\vec{YX} = \vec{OX} - \vec{OY}$.

(ii) Check if $\vec{AX}$ is // to $\vec{YX}$. If it is, then *A*, *X* and *Y* are collinear.

(d) $\vec{XZ} = \vec{OZ} - \vec{OX}$.

(e) Find *XY*, *YZ* and *XZ*.

(f) ΔOAB is similar to ΔXYZ .

$$\Rightarrow \left(\frac{AB}{YZ}\right)^2 = \frac{\text{area of } \Delta OAB}{\text{area of } \Delta XYZ}.$$

Solution

(a) (i) $\vec{DO} = \mathbf{a}$

(ii) $\vec{AB} = \vec{AO} + \vec{OB}$
$= \mathbf{b} - \mathbf{a}$

(iii) $\vec{DB} = \vec{DO} + \vec{OB}$
$= \mathbf{b} + \mathbf{a}$

(b) $|\mathbf{a}| = OA$
$|\mathbf{b}| = OB$
$|\mathbf{b} - \mathbf{a}| = AB$

OAB is an equilateral Δ since *ABCDEF* is a regular hexagon.

$\therefore \quad |\mathbf{a}| = |\mathbf{b}| = |\mathbf{b} - \mathbf{a}|$

(c) (i) (a) $\vec{AX} = \vec{OX} - \vec{OA}$
$= \mathbf{a} + \mathbf{b} - \mathbf{a}$
$= \mathbf{b}$

(b) $\vec{YX} = \vec{OX} - \vec{OY}$
$= \mathbf{a} + \mathbf{b} - (\mathbf{a} - 2\mathbf{b})$
$= 3\mathbf{b}$

(ii) *Y*, *A* and *X* are collinear.

(d) $\vec{XZ} = \vec{OZ} - \vec{OX}$
$= \mathbf{b} - 2\mathbf{a} - (\mathbf{a} + \mathbf{b})$
$= -3\mathbf{a}$

(e) $\vec{XZ} = -3\mathbf{a}$
$XZ = |-3\mathbf{a}|$
$= 3|\mathbf{a}|$

$\vec{YZ} = \vec{OZ} - \vec{Y}$
$= \mathbf{b} - 2\mathbf{a} - (\mathbf{a} - 2\mathbf{b})$
$= 3\mathbf{b} - 3\mathbf{a}$

$YZ = |3\mathbf{b} - 3\mathbf{a}|$
$= 3|\mathbf{b} - \mathbf{a}|$

$\vec{XY} = -3\mathbf{b}$
$XY = |-3\mathbf{b}|$
$= 3|\mathbf{b}|$

Since $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{b} - \mathbf{a}|$, $\therefore$ *XYZ* is equilateral. (shown)

(f) $\dfrac{\text{Area of } \Delta OAB}{\text{Area of } \Delta XYZ} = \left(\dfrac{1}{3}\right)^2$ d
$= \dfrac{1}{9}$

23 *(J2006/P2/Q11 b)*

Questions are not shown in Preview

Question 23

Thinking Process

(b) (i) $PR // PQ \Rightarrow$ gradients are equal.

(ii) $\overrightarrow{PU} = \overrightarrow{PQ} + \overrightarrow{QU}$.

(iii) $\overrightarrow{QU} = \frac{1}{2}\overrightarrow{QS}$. Find *k*.

Solution

(b) (i) Since *R* lies on *PQ*,
$\Rightarrow PR // PQ$
$\Rightarrow$ gradient of *PR* = gradient of *PQ*
$\Rightarrow \dfrac{-6}{h} = \dfrac{-9}{3}$
$\Rightarrow -6 = -3h$
$h = 2$

(ii) $\overrightarrow{PU} = \overrightarrow{PQ} + \overrightarrow{QU}$
$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} + \begin{pmatrix} 7 \\ 2 \end{pmatrix}$
$= \begin{pmatrix} 10 \\ -7 \end{pmatrix}$

(iii) Since *U* is the mid-point of *QS*,
$\Rightarrow \overrightarrow{QU} = \frac{1}{2}\overrightarrow{QS}$
$\Rightarrow \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \frac{1}{2}\left(\overrightarrow{PS} - \overrightarrow{PQ}\right)$
$\Rightarrow \begin{pmatrix} 14 \\ 4 \end{pmatrix} = \begin{pmatrix} 17 \\ k \end{pmatrix} - \begin{pmatrix} 3 \\ -9 \end{pmatrix}$
$\Rightarrow \begin{pmatrix} 14 \\ 4 \end{pmatrix} = \begin{pmatrix} 14 \\ k+9 \end{pmatrix}$
$k + 9 = 4$
$\therefore k = 4 - 9$
$= -5$

24 *(D2006/P1/Q12)*

Questions are not shown in Preview

A

Question 24

Thinking Process

(a) $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$. $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$.

(b) Show $\overrightarrow{OP} = k\overrightarrow{BA}$.

(c) Ratio = 3 : 2.

Solution

(a) $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

$= 2\mathbf{c} + \mathbf{a}$

$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= 4\mathbf{a} - 2\mathbf{c} - \mathbf{a}$

$= 3\mathbf{a} - 2\mathbf{c}$

(b) $\overrightarrow{OP} = 2\mathbf{a} - \frac{4}{3}\mathbf{c}$

$= \frac{2}{3}(3\mathbf{a} - 2\mathbf{c})$

$= \frac{2}{3}\overrightarrow{BA}$

Since $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{BA}$, $\therefore$ $\overrightarrow{OP}$ is parallel to $\overrightarrow{BA}$.

(c) $\dfrac{\text{area of } \Delta OBA}{\text{area of } \Delta OPA} = \dfrac{3}{2}$

J u n e 2 0 0 9

PAPER 1

means " before that, do this ! "

Answer **all** *questions.*

Neither Electronic Calculators Nor Mathematical Tables May Be Used In This Paper.

1 *Topic: 1 Numbers*

Question 1

Thinking Process

(a) Recall BODMAS rules.
(b) Multiply by 100.

Solution

(a) $17 - 5 \times 3 + 1$

$= 17 - 15 + 1$

$= 3$ **Ans.**

(b) $0.82 \times 100 = \frac{82}{100} \times 100 = 82\%$ **Ans.**

2 *Topic: 1 Numbers*

Questions are not shown in Preview

Question 2

Thinking Process

(a) Evaluate the given expression.
(b) Take LCM and simplify.

Solution

(a) $\frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$ **Ans.**

(b) $\frac{3}{4} - \frac{2}{3}$

$= \frac{9-8}{12} = \frac{1}{12}$ **Ans.**

3 *Topic: 1 Numbers*

Questions are not shown in Preview

Question 3

Thinking Process

(a) Think of numbers between 10 and 100 whose cube roots are whole numbers.
(b) Recall, prime numbers are whole numbers that cannot be exactly divided by any number except 1 and themselves.

Solution

(a) The two cube numbers are 27 and 64 **Ans.**

(b) The two prime numbers are 31 and 37 **Ans.**

4 *Topic: 4 Algebraic Expressions and Manipulations*

Question 4

Thinking Process

(a) Recall $a^2 - b^2 = (a+b)(a-b)$
(b) Apply the formula given in part (a).

Solution

(a) $x^2 - y^2 = (x+y)(x-y)$ **Ans.**

(b) $102^2 - 98^2$

$= (102+98)(102-98)$

$= (200)(4) = 800$ **Ans.**

November 2 0 0 9

PAPER 2

means " before that, do this ! "

Section A [52 marks]

Answer **all** *the questions in this section.*

1 *Topic: 5*

Questions are not shown in Preview

Question 1

Thinking Process

(a) Write 8 in index form.
(b) Expand and solve for *p*.
(c) Make a common denominator on the left hand side and solve.
(d) Apply quadratic formula.

Solution

(a) $2^y = 8$

$2^y = 2^3$

$\therefore \quad y = 3$ **Ans.**

(b) $3p + 4 = 8 - 2(p - 3)$

$3p + 4 = 8 - 2p + 6$

$3p + 2p = 8 + 6 - 4$

$5p = 10$

$p = 2$ **Ans.**

(c) $\dfrac{18}{q} - \dfrac{16}{q+2} = 1$

$\dfrac{18(q+2) - 16q}{q(q+2)} = 1$

$18q + 36 - 16q = q(q+2)$

$2q + 36 = q^2 + 2q$

$q^2 = 36$

$q = \pm 6$ **Ans.**

(d) $5x^2 + x - 7 = 0$

Applying quadratic formula,

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-7)}}{2(5)}$$

$$= \frac{-1 \pm \sqrt{141}}{10}$$

$$x = \frac{-1 + \sqrt{141}}{10} \quad \text{or} \quad x = \frac{-1 - \sqrt{141}}{10}$$

$$= 1.0874 \quad \text{or} \quad x = -1.2874$$

$\therefore \quad x = 1.09$ or -1.29 (to 2 dp) **Ans.**

2 *Topic: 9*

Questions are not shown in Preview

Question 2

Thinking Process

(a) (i) *ABCD* is a rectangle with *AP* = *CR*.
(ii) Prove that *BQ* = *SD*. Observe that triangles are congruent by SAS property.
(iii) *PB* is parallel to *DR*, $\angle BPR = \angle DRP$, and $\angle BPQ = \angle DRS$.
(b) Note that *PQ* is parallel to *SR*.

Solution

(a) (i) Given that $AB = DC$ and $AP = RC$

$\therefore \quad PB = AB - AP$

$= DC - RC$

$= RD$ **Shown.**

(ii) Given that, $BC = AD$ and $QC = AS$

$\Rightarrow \quad BQ = BC - QC$

$= AD - AS$

$= DS$

$\therefore \quad BQ = DS$

from part (a) (i): $PB = RD$

also $P\hat{B}Q = R\hat{D}S = 90°$

$\therefore \quad \Delta PBQ \equiv \Delta RDS$ (SAS) **Shown.**

(iii) $ABCD$ is a rectangle, therefore PB is parallel to DR.

$\Rightarrow \quad B\hat{P}R = D\hat{R}P$ (alternate $\angle$s)

and $B\hat{P}Q = D\hat{R}S$ $(\Delta PBQ \equiv \Delta RDS)$

now,

$B\hat{P}Q + R\hat{P}Q = B\hat{P}R$

$R\hat{P}Q = B\hat{P}R - B\hat{P}Q$

$= D\hat{R}P - D\hat{R}S$

$= P\hat{R}S$

$\therefore \quad R\hat{P}Q = P\hat{R}S$ **Shown.**

(b) From (a) (iii), $R\hat{P}Q = P\hat{R}S$

$\Rightarrow$ PQ is parallel to SR

$\therefore$ $PQRS$ is a parallelogram. **Ans.**

3 *Topic: 14*

Questions are not shown in Preview

Question 3

Thinking Process

(a) Apply $\sin\theta = \frac{\text{opp}}{\text{hyp}}$.

(b) Apply $\sin\theta = \frac{\text{opp}}{\text{hyp}}$.

(c) (i) To find angle *BMC* find angle *MBA.*

(ii) Apply $\sin M\hat{B}C = \frac{\text{opp}}{\text{hyp}}$.

Solution

(a) $\sin 15° = \frac{d}{50}$

$d = \sin 15° \times 50$

$= 12.941 \approx 12.9$ m (3sf) **Ans.**

(b) In ΔAMB,

$\sin 15° = \frac{10}{AB}$

$AB = \frac{10}{\sin 15°}$

$= 38.637 \approx 38.6$ m (3sf) **Ans.**

(c) (i) In ΔAMB, $A\hat{M}B = 90°$

$\therefore \quad M\hat{B}A = 90° - 15° = 75°$

Point C is nearest to point M,

$\therefore \quad M\hat{C}B = 90°$

In ΔBMC,

$B\hat{M}C = 90° - M\hat{B}C$

$= 90° - 75°$

$= 15°$ **Ans.**

(ii) In ΔBMC,

$\sin M\hat{B}C = \frac{CM}{BM}$

$\sin 75° = \frac{CM}{10}$

$CM = \sin 75° \times 10$

$= 9.659 \approx 9.66$ m (3sf) **Ans.**